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# The effects of income redistribution on sectorial aggregate demand curves

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by

Plinio Mario Nastari

A Thesis Submitted to the  
Graduate Faculty in Partial Fulfillment of the  
Requirements for the Degree of  
MASTER OF SCIENCE  
Department: Economics  
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Signatures have been redacted for privacy

Iowa State University  
Ames, Iowa  
1981

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## PREFACE

The major concern of this study is the formulation of a methodology for the analysis of the effects of income redistribution on sectoral aggregate demand curves. Its conception germinated after I explored the works of William Cline on the potential effects of income redistribution on economic growth in Latin American countries. In his attempt to estimate the new levels of quantity demanded for each sector in those economies, Cline uses a methodology which enables him to estimate only the shifts of the sectoral aggregate demand curves at the equilibrium price levels observed before income redistribution.

I contend that this information is not always conclusive, since the new equilibrium levels of price and quantity in each sector depend also on the behavior of the sectoral aggregate supply curves. Cline's approach yields valid conclusions on the effects of income redistribution on aggregate demands only for those sectors



which have perfectly elastic aggregate supplies.

The present work should be regarded as an early attempt to set together my thoughts on this issue. It does not include any estimations using real data. A revision of Cline's work using this new methodology is perhaps infeasible, for it would require access to budget studies for Latin American countries realized in many different years, something inexistent at this time. Nevertheless, it could be easily applied for studying those countries where budget studies are conducted more often.

This study is divided into two parts.

A study without reflection is a waste of time. Reflection without study is dangerous. Therefore, in Part I I conduct a review of the literature of Utility Theory (Chapter I), of demand curves (Chapter II), and of the relation between income distribution and demand curves (Chapter III).

I present my proposal in Part II, with a scanning of Cline's work dominating Chapter IV. Chapter V, a long one, contains the exposition of the new methodology.

I want to express my most grateful thanks to the American Chambers of Commerce for Brazil in São Paulo and Rio de Janeiro, and to National Distillers do Brasil Ltda., for sponsoring my graduate studies at Iowa State University from September of 1979 through February of 1981. My thanks in a special way go to Mr. David Wicker, who truly believes that knowledge is one of the few instruments of production that are not subject to diminishing marginal returns.

Most grateful thanks also to President Ernesto Geisel for being the so-special patron of my scholarship. His brilliant career and unmatched dedication to Brazil have been sources of inspiration and renewed strength.

Special thanks go to the Department of Economics at Iowa State University for providing me a teaching assistantship from March through May of 1981, which enabled the completion of this work. Dr. James Stephenson has been a remarkable major professor -- although many times I would prefer to call him 'coach' -- giving me the valuable ease to develop my studies and research, and always expressing most concern through his wise advice.



The suggestions and comments of Drs. Harvey Lapan and Dennis Starleaf have also been instrumental for the satisfactory completion of this work.

To my family in Brazil, my thanks for bearing this extended absence from home.

Thanks also to Susan Snyder and Joan Burg from the Media Resources Center at I. S. U. who demonstrated a relentless helpfulness drawing the graphs.

At the expenses of my academic naiveté, I add that this is an honest paper. I truly believe in every idea stated hereafter.

The fruit derived from labor is really the sweetest of pleasures.

Plinio Mario Nastari

Ames, Iowa

April, 1981.

PART I.

REVIEW OF THE LITERATURE

## CHAPTER I.

## DEVELOPMENTS OF UTILITY THEORY

This chapter will focus on the essential and most remarkable achievements of Utility Theory, which occurred during the period from 1850 to 1940. We will omit all applications of Utility Theory, in particular those to Welfare Economics, other than the ones related to the derivation of demand curves. To this respect, we will give emphasis to what we consider are the two major breakthroughs in the development of Utility Theory, one by Edgeworth and the other by Fisher and Pareto, since today they constitute the base for the modern approach to Consumer Choice Theory. More extensive reviews of the developments of Utility Theory can be found in the works of Stigler,<sup>1</sup> Houthakker<sup>2</sup> and Blaug.<sup>3</sup>

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<sup>1</sup>George Stigler, "The Development of Utility Theory," Journal of Political Economy 58(1950):307-27, 373-96.

<sup>2</sup>H. S. Houthakker, "The Present State of Consumption Theory," Econometrica 29(1961):704-40.

<sup>3</sup>Mark Blaug, Economic Theory in Retrospect (Homewood, Ill.: Irwin, 1969).

### The Water-Diamond Paradox

Since Adam Smith, economic theory has been concerned with the behavior of demand, trying to connect it to the structure of consumer desires. For one thing, it seemed natural to associate demand behavior to the utility of a commodity, as the consumer perceives it. Therefore, the main problem faced by early economists was to find a sound reason for why the price paid for a commodity was not always related to the utility associated with its consumption. Water and diamonds were found to be classical examples of such paradox, since water, which is essential to life and therefore of great utility, commands often a very low price, whereas diamonds, whose utility was said to be less than that of water, are notoriously expensive.

This apparent paradox was explained by an analysis which was the focal point of the economic literature at the turn of the century. It was argued that the price of a commodity was determined not by its total, but by its marginal utility.

Jevons,<sup>1</sup> Menger<sup>2</sup> and Walras,<sup>3</sup> who were called by Stigler<sup>4</sup> the three founders of the Utility Theory (better known today as the marginal utility theorists), independently and simultaneously arrived at positions similar in the main and sometimes in detail.<sup>5</sup>

### The Marginal Utility Theorists

The discovery that price and marginal utility are related concepts came from the realization by the marginal utility theorists that if a rational consumer holds  $n$  units of a certain commodity  $X$ , and if the marginal utility that he can obtain from the possession of an additional unit of  $X$  is larger than its price, he can increase

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<sup>1</sup>W. S. Jevons, The Theory of Political Economy (London: Macmillan, 1871).

<sup>2</sup>Carl Menger, Grundsätze der Volkswirtschaftslehre (Vienna: Braumüller, 1871).

<sup>3</sup>L. Walras, Éléments d'économie politique pure (Lausanne: Carbay, 1874), published in English as Elements of Pure Economics (London: Allen and Unwin, 1954).

<sup>4</sup>Stigler, "The Development of Utility Theory," p. 316.

<sup>5</sup>Ibid., p. 315.

his welfare by purchasing this additional unit of X. He may repeat this operation with advantages up to the point where price equals marginal utility. This is so, simply because he receives more value than he gives up in such exchanges.

Even though this was the first explicit condition for utility maximization ever to be drawn, the marginal utility theorists carried their analysis considerably further.<sup>1</sup> They also consolidated the position of the concept of diminishing marginal utility in Economic Theory - the more we possess of a commodity, the less we value an additional unit of it.

Gossen<sup>2</sup> was the first author to formulate clearly the Law of Diminishing Marginal Utility, and to apply it to individual acts of consumption.<sup>3</sup> However, his

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<sup>1</sup>All the realizations of the marginal utility theorists are analyzed in great detail in R. S. Howey, The Rise of the Marginal Utility School 1870-1889 (Lawrence, Ks.: University of Kansas Press, 1960).

<sup>2</sup>Heinrich H. Gossen, Entwicklung der Gesetze des menschlichen Verkehrs, und der daraus fließenden Regeln für menschliches Handeln, 3<sup>rd</sup> ed. (Berlin: Prager, 1927), first published in 1854.

<sup>3</sup>See Blaug, Economic Theory in Retrospect, p. 280.



work attracted no attention at the time of its publication, probably because Gossen's method of exposition was such that few readers, even now, could follow his arguments.<sup>1</sup> This might be, perhaps, the reason why Jevons cited Jennings<sup>2</sup> instead of Gossen as his authoritative source when he wrote the Law of Diminishing Marginal Utility as an appeal ". . . to the physiological law that the strength of the response to a stimulus diminishes with each repetition of that stimulus within some specified time period."<sup>3</sup>

The Law of Diminishing Marginal Utility was the long-sought explanation for the negative slope which is alleged to characterize most simple demand curves. The plain reason for that comes from the fact that if the marginal utility of a commodity falls when the consumer purchases more of it, he can only be induced to buy more

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<sup>1</sup>See Stigler, "The Development of Utility Theory," p. 314.

<sup>2</sup>Richard Jennings, Natural Elements of Political Economy (London: Longman, Brown, Green & Longmans, 1855), pp. 98-99, 119.

<sup>3</sup>See Blaug, Economic Theory in Retrospect, p. 284.

of the good by a fall in its price.

Furthermore, the marginal utility theorists believed in additive utility functions, and in a cardinal measure of utility. Both issues were to be attacked a few years later by their fellow economists.

### Edgeworth

The first attack on the marginal utility theorists concerned their concept of additive utility functions. It is very interesting and fruitful to analyze how the specification of the utility function evolved, and what the theoretical implications are for the two major types of specification, the additive and the generalized.

Gossen<sup>1</sup> was the first to give a systematic contribution to the subject when he assumed that consumer's preferences could be represented by a sum of quadratic expressions in the quantities consumed, all cross-product

---

<sup>1</sup>Gossen, Entwicklung.



terms being zero.<sup>1</sup>

The marginal utility theorists, in turn, treated the utility of a commodity as a function only of its quantity, corresponding to the additive specification of utility functions. Therefore, if  $X_1, X_2, X_3, \dots$  were the commodities, the individual's total utility could be written as (explicitly by Jevons and Walras, and implicitly by Menger):<sup>2</sup>

$$U = f(x_1) + g(x_2) + h(x_3) + \dots$$

The assumptions of diminishing marginal utility provided the sufficient second-order equilibrium conditions for utility maximization for this specification, which obviously did not involve cross-product second-order partial derivatives.<sup>3</sup>

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<sup>1</sup>See Houthakker, "The Present State of Consumption Theory," p. 705; P. A. Samuelson, Foundations of Economic Analysis (Cambridge: Harvard University Press, 1947), p. 93.

<sup>2</sup>See Stigler, "The Development of Utility Theory," p. 322.

<sup>3</sup>However, it is not necessary that we have diminishing marginal utility for each commodity to have indifference curves convex to the origin. Appendix I deals with this, and other related subjects.

Edgeworth<sup>1</sup> destroyed this pleasant simplicity when he wrote the total utility function as a generalized function of all quantities:

$$U = U(x_1, x_2, x_3, \dots)$$

He sustained the conditions of diminishing marginal utility and imposed no restrictions on cross-product second-order partial derivatives. Eight years later, Auspitz and Lieben<sup>2</sup> would also adopt Edgeworth's proposed specification for the utility function.<sup>3</sup>

Quoting Whitehead's apothegm, "everything of importance has been said before by somebody who did not discover it."<sup>4</sup> So it was with Edgeworth, who never realized the importance and extension of his contribution.

<sup>1</sup>F. Y. Edgeworth, Mathematical Physics (London: Kegan Paul, 1881).

<sup>2</sup>R. Auspitz and R. Lieben, Untersuchungen über die Theorie des Preises (Leipzig: Duncker and Humblot, 1889).

<sup>3</sup>See Houthakker, "The Present State of Consumption Theory," p. 705.

<sup>4</sup>Taken from Blaug, Economic Theory in Retrospect, p. 283.

Under the generalized function, diminishing marginal utility is no longer a necessary nor sufficient condition for utility maximization, subject to a budget constraint. Also, this departure from the earlier concept of an additive utility function, set by the marginal utility theorists, gave rise to the mathematical proof of the cases when ordinary demand curves may have positive slopes and Engel curves may have negative slopes.<sup>1</sup> These achievements were to be confirmed later by Slutsky, through the formulation of his famous equation, to be presented in the next section.

#### Fisher and Pareto

Gossen, Jevons, Menger, Walras, Edgeworth, Marshall,<sup>2</sup> and Auspitz and Lieben, all viewed utility as being cardinal. The marginal utility theorists, in

---

<sup>1</sup>In Appendix I we analyze the complete mathematical implications of Edgeworth's generalized utility function.

<sup>2</sup>Alfred Marshall, Principles of Economics (London: Macmillan, 1890).

particular, were dissatisfied with a money measure of marginal utility, because the value of money is not constant overtime. Therefore, measuring marginal utility in terms of money is like "calculating length with a rubber ruler which stretches as we measure."<sup>1</sup> They proposed thereafter that marginal utility should be measured in its own subjective units, something that has been called utils. This cardinal measure of utility was to be under fire a few years later.

In the 1890s, Fisher<sup>2</sup> and Pareto<sup>3</sup> realized that if a utility function reaches a maximum at a certain point, then any order-preserving transformation of that function also reaches a maximum there. They concluded, consequently, that such maximization involves only ordinal properties.

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<sup>1</sup>William J. Baumol, Economic Theory and Operation Analysis (Englewood Cliffs, N.J.: Prentice-Hall, 1977), p. 192.

<sup>2</sup>I. Fisher, "Mathematical Investigations in the Theory of Value and Prices," Transactions of Connecticut Academy of Arts and Sciences 9(1892):1-124.

<sup>3</sup>Vilfredo Pareto, Cours d'économie politique (Lausanne: F. Rouge, 1896-7).

A clean, clear mathematical proof of this notable discovery is presented by Henderson and Quandt.<sup>1</sup>

If after Marshall<sup>2</sup> utility surface was a common designation for the locus of points each of which represents a collection of commodities such that the consumer experiences the same level of satisfaction at each point (measured in a cardinal sense), after the breakthrough of Fisher and Pareto an indifference curve is known as the locus of points each of which represents a collection of commodities such that the consumer is indifferent among any of these combinations. And an indifference map is the designation for the set of indifference curves for a decision unit. Figure 1 presents a set of indifference curves for a hypothetical individual in a two-commodity world ( $X_1$  and  $X_2$ ) containing its commonly attributed properties.

The specification of the properties of indiffer-

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<sup>1</sup>James M. Henderson and Richard E. Quandt, Micro-economic Theory, A Mathematical Approach (New York: McGraw-Hill, 1958), pp. 20-22.

<sup>2</sup>Marshall, Principles of Economics.

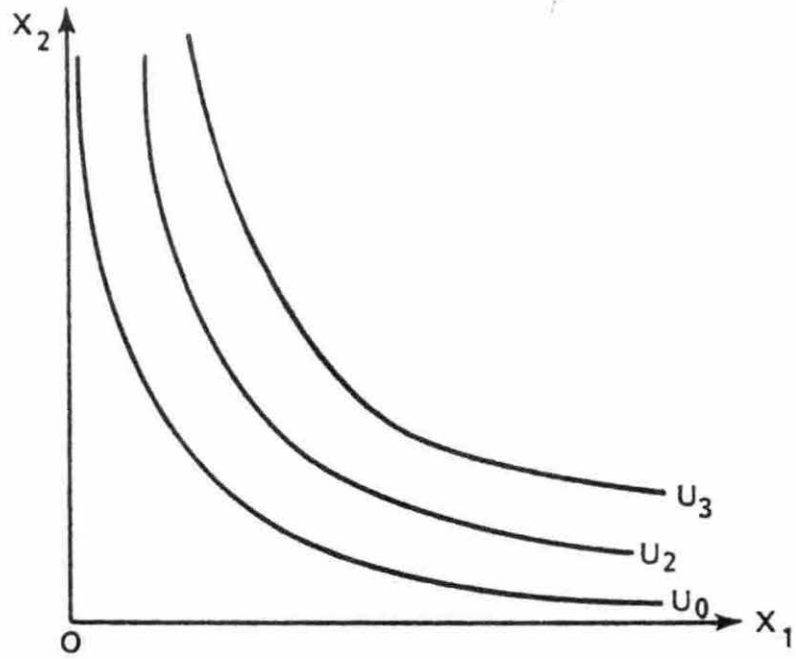


Figure 1. Utility surfaces or indifference curves



ence curves evolved very little with time. Baumol<sup>1</sup> teaches us how to elegantly deduce the four properties of indifference curves, which we reproduce below.

Property I

An indifference curve which lies above and to the right of another represents preferred combinations of commodities.

Property II

Indifference curves have a negative slope.

Property III

Indifference curves can never meet or intersect.

Property IV

The absolute slope of an indifference curve diminishes toward the right, so that the curve is said to be convex to the origin.

Property IV of indifference curves, which is a direct descendant from the Law of Diminishing Marginal

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<sup>1</sup>Baumol, Economic Theory, pp. 197-8.

Utility, which was first formulated by Gossen, became the base for Slutsky's findings in regard to the negative slope of compensated demand curves<sup>1</sup> (the compensated own-price derivatives are always negative), also known as the Slutsky inequality:

$$\left(\frac{dX_i}{dP_i}\right)_{U=\text{const}} < 0$$

Slutsky also derived two other expressions which are departures from Fisher's and Pareto's generalization. One is the Slutsky equality, according to which the compensated cross-price derivatives are pairwise equal to each other:

$$\left(\frac{dX_i}{dP_j}\right)_{U=\text{const}} = \left(\frac{dX_j}{dP_i}\right)_{U=\text{const}}$$

The other is the Slutsky equation, which identifies the substitution and income effects of a change in price over quantity demanded:

---

<sup>1</sup>E. E. Slutsky, "Sulla Teoria del Bilancio del Consumatore," Giornale degli Economisti 51(1915):1-26.



$$\frac{dX_i}{dP_i} = \left(\frac{dX_i}{dP_i}\right)_{U=\text{const}} - X_i \left(\frac{dX_i}{dY}\right)_{\text{Prices}=\text{const}}$$

This equation, like Edgeworth's generalized utility function, enables us to understand the case of inferior goods,<sup>1</sup> with the added advantage that we are able to verify that the ordinary or classical demand curve will have a positive slope when the income effect is negative and sufficiently large to overcome the substitution effect.

Therefore, we stress the fact that Edgeworth's generalized utility function and Fisher's and Pareto's ordinal measure of utility were the starting points for the later developments toward the understanding of Engel curves and demand curves in all their possible forms.

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<sup>1</sup>See Appendix I.

Alternative Approaches to the  
Study of Consumer Preferences

After Fisher and Pareto, the generalizations on the concept of utility had so undermined the reality of its concept, which to Edgeworth was "as real as his morning jam,"<sup>1</sup> that economists like Cassel<sup>2</sup> and Allen<sup>3</sup> started to formulate alternative approaches to the study of preferences.

Cassel was probably the most radical of them all, declaring that economics should start out of demand functions rather than from utility functions since for him they belonged to psychology.<sup>4</sup> Notwithstanding, he had to rely indirectly on the concept of preferences when he

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<sup>1</sup>Houthakker, "The Present State of Consumption Theory," p. 706.

<sup>2</sup>Gustav Cassel, Theoretische Sozialökonomie (Leipzig: Scholl, 1918).

<sup>3</sup>R. G. Allen, "A Reconsideration of the Theory of Value, II," Economica 1(1934): 196-219.

<sup>4</sup>Houthakker, "The Present State of Consumption Theory," p. 706.

found it necessary to attribute to the demand functions the property of homogeneity of degree zero in income and prices, that is completely arbitrary unless demand functions are held to reflect underlying preferences.<sup>1</sup>

Allen tried a different approach to avoid the use of the utility concept, relying instead on the marginal rates of substitution between commodities. By doing that he implicitly admitted only comparisons between bundles of commodities that are infinitesimally close to each other.

However, neither Cassel's nor Allen's approaches found general acceptance. In the remarkable works of Hicks<sup>2</sup> and Wold<sup>3</sup> the fundamental concepts are those of an ordinal measure of utility and of indifference curves, following Slutsky's path.

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<sup>1</sup>Houthakker, "The Present State of Consumption Theory," p. 706.

<sup>2</sup>J. R. Hicks, "A Reconsideration of the Theory of Value, I," Economica 1(1934): 52-75; J. R. Hicks, Value and Capital (New York: Oxford University Press, 1939).

<sup>3</sup>H. Wold, "A Synthesis of Pure Demand Analysis," Skandinavisk Aktuarietidskrift 26(1943): 85-118, 220-263, and 27(1944): 69-120.

Perhaps, the strongest alternative approach to the concept of utility functions was proposed by Samuelson, who intended to "start anew in direct attack upon the problem, dropping off the last vestiges of the utility analysis,"<sup>1</sup> with the introduction of revealed preferences.<sup>2</sup> Samuelson's objective was to determine sufficient conditions for demand functions that could be expressed in terms of individual price-quantity situations, rather than in terms of derivatives of demand functions.<sup>3</sup>

Although Samuelson's proposition was epochal and still today receives a lot of attention, economists in general agree that utility and indifference curves are strongly established concepts in Consumer Choice Theory, and are most helpful instruments of analysis to reflect

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<sup>1</sup>P. A. Samuelson, "A Note on the Pure Theory of Consumer's Behaviour," Economica 5(1938):61-71, 353-4, p. 62.

<sup>2</sup>For a good summary of the Theory of Revealed Preferences see Houthakker, "The Present State of Consumption Theory," pp. 706-10.

<sup>3</sup>Ibid., p. 706.



consumer preferences, especially in the derivation of various definitions of demand curves.

## CHAPTER II.

THE MOST IMPORTANT  
DEFINITIONS OF DEMAND CURVES

In Part II, we will be referring to demand curves extensively; therefore, it is utmost necessary that we understand and review the available definitions of demand curves, so that when we refer to a demand curve we know which definition we are talking about.

However, before we proceed in our investigation, we must remember the difference between a demand function and a demand curve. Baumol gives a good treatment to this subject:

. . . demand is a function of many variables such as price, advertising, and decisions relating to competing and complementary products. The relationship which describes this entire many variable interconnection is called the demand function. By contrast, the demand curve deals only with two of these variables, price and quantity demanded, and ignores the others, or rather, assumes that their values are held constant.<sup>1</sup>

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<sup>1</sup>Baumol, Economic Theory, p. 182.

Other economists<sup>1</sup> have tackled the same issue with no divergences from Baumol.

Another concept which will be useful to state precisely at this time is that of Engel curve.<sup>2</sup> This curve shows the different quantities of a particular good that the consumer will take at various levels of income, other things equal (particularly prices).

Calling attention to the difference between demand functions and demand curves, we anticipate that all definitions of demand curves differ basically on what is held constant in the demand function, given a change in price.

The first definition, we will be looking at, is the classical, ordinary, or Marshallian demand curve. This curve is also known as the constant-money-income

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<sup>1</sup>See for example George H. Haines Jr., "Overview of Economic Models of Consumer Behavior," in Consumer Behavior: Theoretical Sources, eds. Scott Ward and Thomas S. Robertson (Englewood Cliffs, N.J.: Prentice-Hall, 1973); I. F. Pearce, "Total Demand Curves and General Equilibrium," Review of Economic Studies 20(1953):216-221.

<sup>2</sup>We have already referred to Engel curves in Chapter I.

demand curve, a denomination which speaks up for itself.

Secondly, we will inspect the trader's demand curve, as it was called by L. Walras.<sup>1</sup> This curve is applicable to a person who in a two-commodity world goes to market with a given stock of the two goods and may purchase more of either, depending on the going relative market price for the two commodities.

The third in the list is the compensated demand curve, also called constant-real-income demand curve. This curve has found in Friedman its fierce defender, although its origins go back to Slutsky.

The fourth and last definition we will analyze is the production-frontier demand curve, a concept introduced by Bailey as a response to Friedman's apology of the latter definition.

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<sup>1</sup>L. Walras, Elements of Pure Economics (London: Allen and Unwin, 1954), first published as Éléments d'économie politique pure (Lausanne: Carbay, 1874).



### The Classical Demand Curve

The classical derivation of a demand curve was first formulated by Marshall<sup>1</sup>, being further perfected by Slutsky<sup>2</sup>, who incorporated into it Fisher's and Pareto's concept of an ordinal measure of utility.<sup>3</sup>

Marshall's main argument was that a consumer with a given money income is confronted with a market for consumption commodities where money prices are given. He assumed, first, that the consumer derives different levels of utility if he consumes different bundles of goods; second, that the consumer is able to determine the level of utility achieved (implying a cardinal measure of utility); and third, that he will spend his money income in such a way as to achieve the maximum level of utility possible. The bundles of commodities yielding the same

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<sup>1</sup>Marshall, Principles of Economics.

<sup>2</sup>Slutsky, "Sulla Teoria del Bilancio del Consumatore."

<sup>3</sup>This matter has been analyzed in Chapter I.

levels of utility could then be grouped and a utility surface be drawn for each level of utility.

The innovation brought up by Slutsky called for the fact that Marshall's utility surface implied a cardinal measure of utility, and that the derivation of the demand curve could still be done under Pareto's framework, which conveys less information since it implies an ordinal measure of utility. Therefore, Figure 1 can also be seen as depicting a set of indifference curves provided that we keep the ordering of the curves and that we do not specify a definite level of utility for each one.

Besides the set of indifference curves, a second analytical instrument is needed for the derivation of a demand curve; the line of attainable combinations. Given the money income available for expenditure and the money prices of both commodities, the line of attainable combinations (also known as budget constraint) will have a constant slope equal to the ratio of prices of the two commodities. This line, which is represented in Figure 2 by the segment MN, is mathematically derived from the

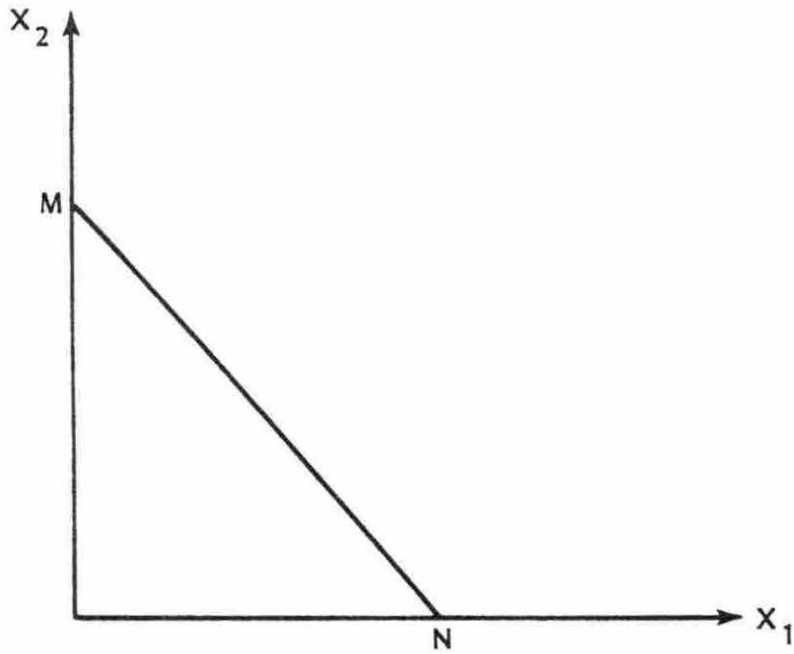


Figure 2. The line of attainable combinations

relation below, and assumes that all income is spent with the two commodities:

$$Y = P_1x_1 + P_2x_2$$

where  $Y$  is the available money income,  $x_i$  is the amount to be purchased, and  $P_i$  is the money price of commodity  $i$ , for  $i = 1, 2$ .

If we superimpose Figures 1 and 2, assuming that the consumer is rational, he will choose that combination of commodities which gives him the maximum level of utility; that is, the combination given by the tangency point between the line of attainable combinations and the highest indifference curve (point A). This step is illustrated in Figure 3.<sup>1</sup>

Assuming everything else constant, if there is an exogenous decrease in the price of commodity  $X_1$  then the line of attainable combinations will shift from MN to MQ, in Figure 4A. Real income for this consumer will go up

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<sup>1</sup>The derivation procedure up to this point will apply to all three other definitions of demand curves, and will not be repeated.

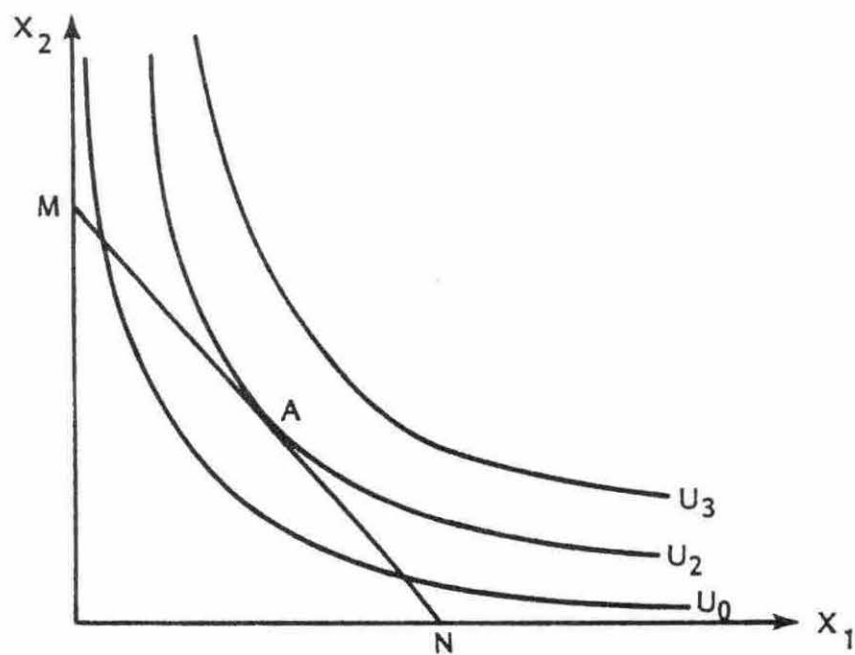


Figure 3. Maximizing utility given a budget constraint

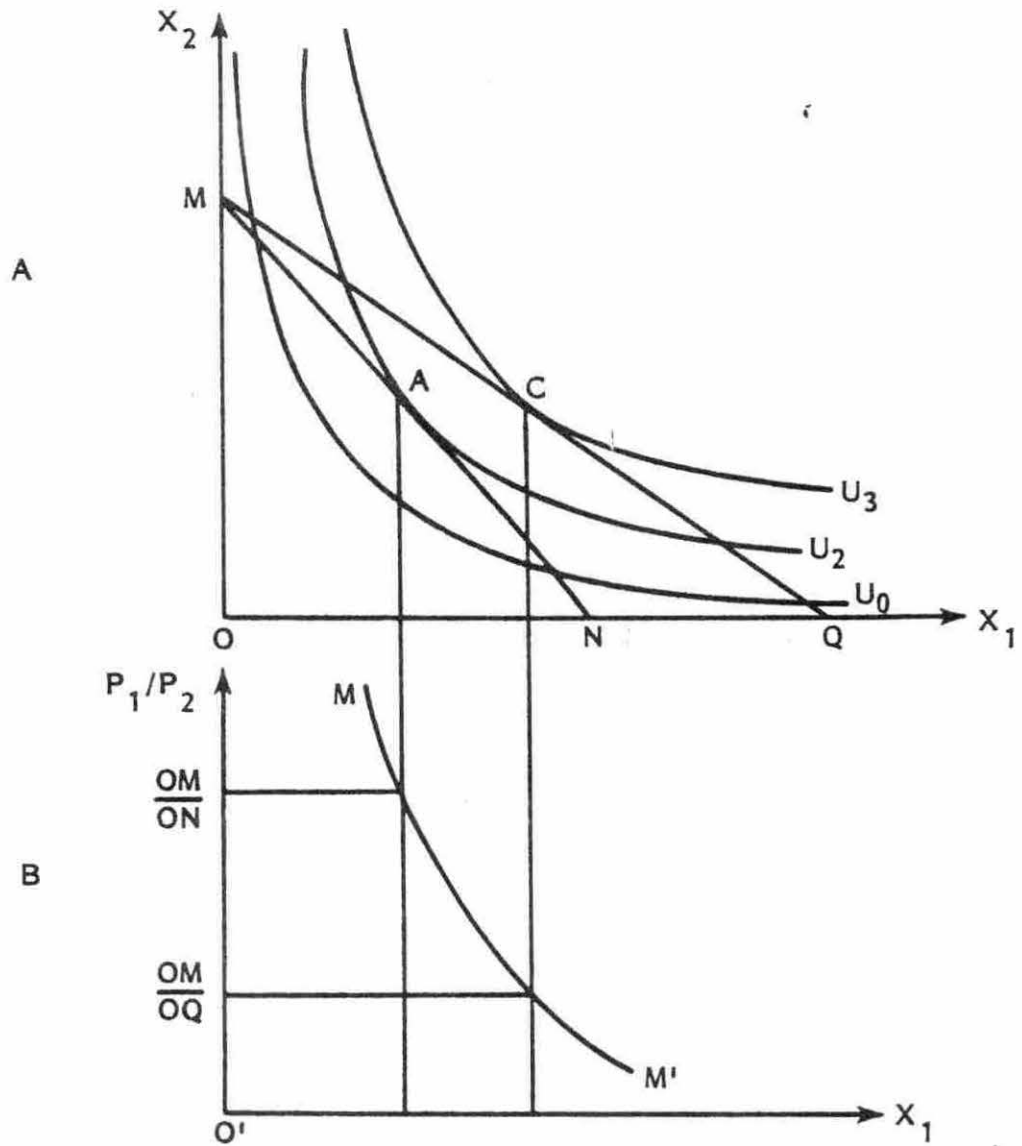


Figure 4. Derivation of the classical demand curve



since the new set of attainable combinations is larger than initially, although his money income is still the same. Under the new budget constraint (MQ) the consumer will choose to consume the combination of commodities that gives him the maximum utility (point C).

Repeating this procedure for many levels of  $P_1$ , holding  $Y$  and  $P_2$  constant, we derive the classical demand curve for commodity  $X_1$ , curve  $MM'$ , as shown in Figure 4B.

The ordinary demand curve is the most commonly used and referred definition of demand curve. Its applications, however, should be limited to those cases when money income is held constant.

#### The Trader's Demand Curve

This definition of demand curve was first formulated by Walras<sup>1</sup>, further adopted by Wicksell<sup>2</sup>, and

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<sup>1</sup>L. Walras, Elements of Pure Economics.

<sup>2</sup>Knut Wicksell, Lectures on Political Economy (New York: Macmillan, 1934), pp. 35-51.

specifically derived by Boulding.<sup>1</sup>

Following Walras's reasoning<sup>2</sup>, let us imagine a market to which some people come holding commodity  $X_1$ , ready to exchange part of it in order to procure commodity  $X_2$ , while others come holding commodity  $X_2$ , ready to exchange part of their  $X_2$  in order to procure commodity  $X_1$ . Since the bidding will have to start at some point or another, Walras introduces the figure of the broker or auctioneer, who will try to set relative prices so as to satisfy everyone's interests. Then, suppose that early enough a general agreement is reached so that no one is left unsatisfied, this final bidding conforming to the equation of exchange:

$$P_1 x_1 = P_2 x_2$$

where  $P_i$  is defined by Walras as "the value in exchange of one unit of  $X_i$ ," for  $i = 1, 2$ . In Walras's own words:

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<sup>1</sup>Kenneth Boulding, "The Concept of Economic Surplus," American Economic Review 35(1945): 851-69.

<sup>2</sup>L. Walras, Elements of Pure Economics, p. 89.

The effective demand for or offer of one commodity in exchange for another is equal respectively to the effective offer of or demand for the second commodity multiplied by its price in terms of the first.<sup>1</sup>

Wicksell's concept of price formation in the open market follows the same argumentation as Walras's. However, the precise derivation of the trader's, or Walrasian demand curve from indifference curves was formulated by Boulding.

Figure 5A presents a set of indifference curves for a single trader (buyer or seller depending on the circumstances), and again, generally any point on indifference curve  $U_n$  is preferred to any point on  $U_{n-1}$ . Suppose now that the trader owns a quantity OR of commodity  $X_1$  and RA of commodity  $X_2$ , such that point A represents his initial position. Given a situation in which he can exchange any amount of either commodity at a given price, to what point will he move? If the relative market price is OM/ON, point A is the utility maximizing

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<sup>1</sup>L. Walras, Elements of Pure Economics, p. 89.

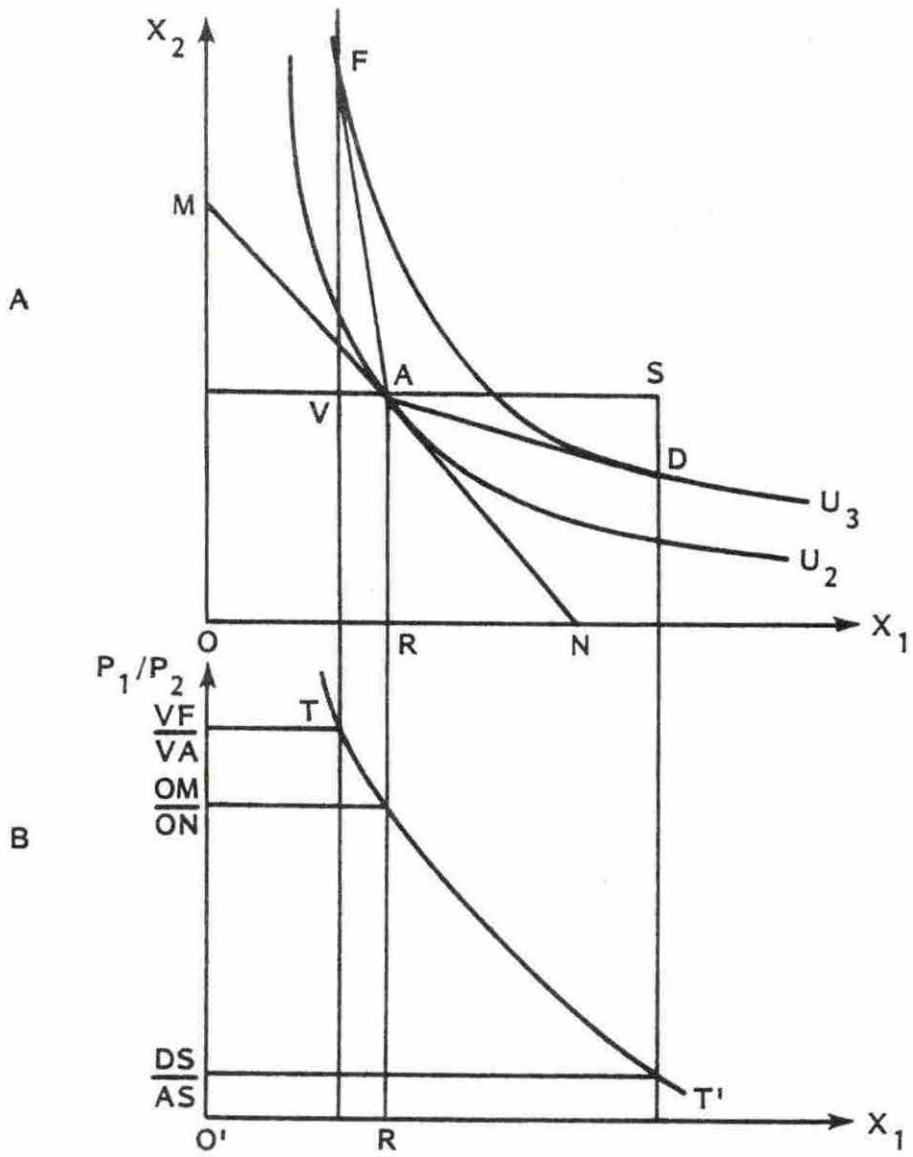


Figure 5. Derivation of the trader's demand curve

point, and he will not change his initial position, the opportunity line being MN. At any other constant price relation it is a straight line through the point A, the slope of which is equal to the market price. Thus, if the price is  $DS/AS$ , the opportunity line will be AD, and the utility maximizing point is D, meaning that the trader is giving up some  $X_2$  in exchange for more  $X_1$ . If the price is  $VF/VA$ , the opportunity line is AF, and the trader is giving up some  $X_1$  in exchange for more  $X_2$ . Repeating this procedure for many price combinations we obtain the trader's demand curve TT'.

This definition of demand curve yields an insight into where money holdings are derived from, since we can say that anybody's money holdings in the marketplace reflects the different combinations of goods that each person owns, valued at market prices. That is, each person essentially exchanges commodities in the marketplace, whether they are hours of labor, grains, currency, or any other commodity. Consequently, we are able to aver that when relative prices of commodities change, there will be also a change in the money holdings (money income) of



individuals possessing those commodities.

### The Compensated Demand Curve

The distinction between the constant-money-income demand curve and the constant-real-income demand curve was first explored by Slutsky, who enlightened us with his famous equation.<sup>1</sup> The derivation of the compensated demand curve is rather simple, and the main idea underlying it is the separation of the total impact of a price change on quantity demanded into two separate effects, substitution and income effects. The substitution effect is what gives rise to the compensated demand curve.

Figures 6A and 6B basically reproduce the same construction of Figures 4A and 4B, such that as the market price of the two commodities is exogenously lowered from  $OM/ON$  to  $OM/OQ$ , given money income constant, the consumer's line of attainable combinations shifts from  $MN$  to  $MQ$ , and the utility maximizing consumption combination moves from

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<sup>1</sup>Slutsky, "Sulla Teoria del Bilancio del Consumatore," see also Chapter I above.



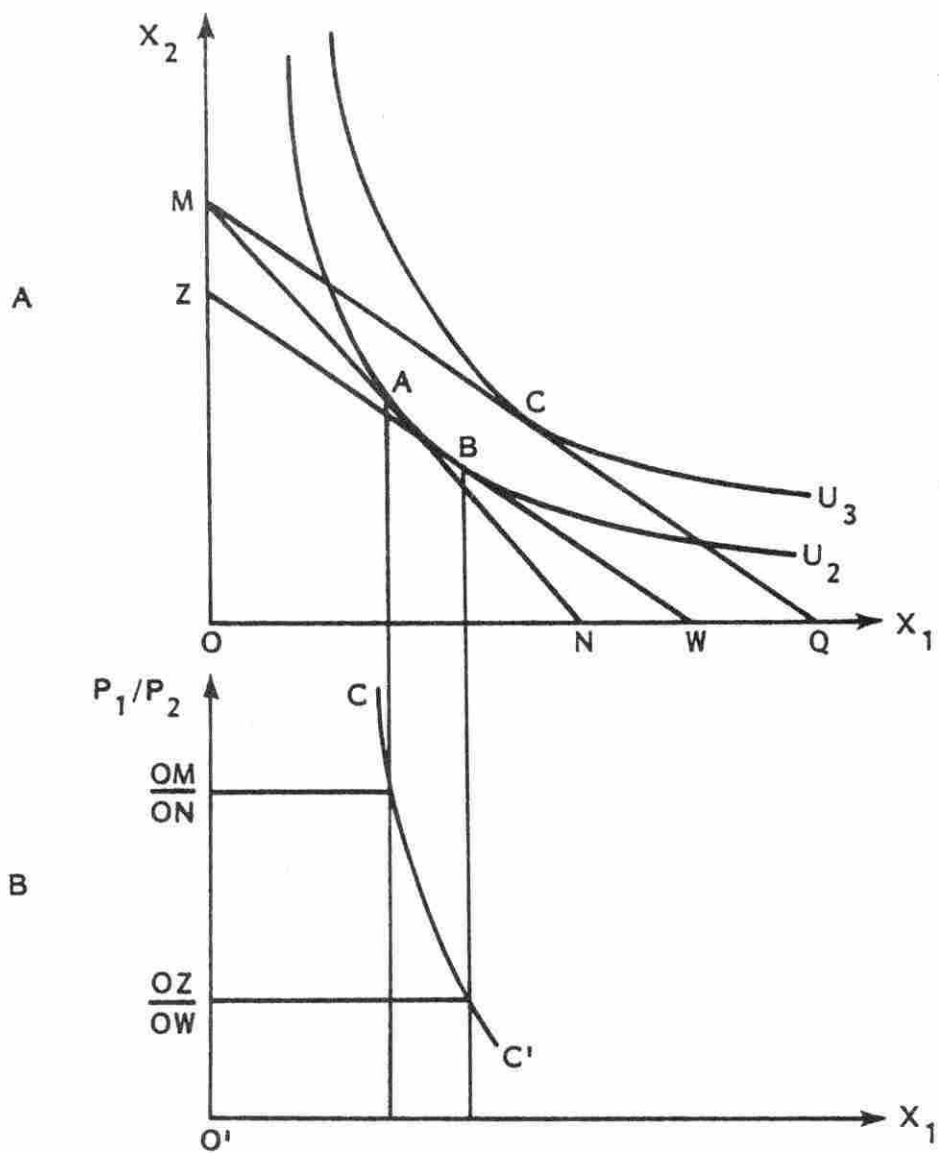


Figure 6. Derivation of the compensated demand curve

A to C.

However, if we assume that real income is constant, we are saying that the consumer will not be able to achieve a higher level of satisfaction, and will still be on the same indifference curve,  $U_2$ . At the new price combination  $OM/OQ = OZ/OW$ , he will no longer maximize his utility at point A, but at B instead. Repeating this procedure for other price combinations, we obtain the compensated demand curve.

Not too much attention was given to the applications of the compensated demand curves until Friedman's notable paper.<sup>1</sup> He argues in it that Marshall did not specify precisely what he meant by the caeteris paribus condition he attached to his definition of demand curve, and that the idea of a constant-money-income demand curve was due more to "other economists . . . (who) constructed a rigorous definition to fill the gap that Marshall left."<sup>2</sup>

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<sup>1</sup>Milton Friedman, "The Marshallian Demand Curve," Journal of Political Economy 57(1949):463-95.

<sup>2</sup>Ibid., p. 463; see also Milton Friedman, Lectures in Price Theory (Chicago: Aldine-Atherton, 1966).

In addition, Friedman insists that Marshall was not speaking so much about money income, but about real income being held constant, and that the attribution to Marshall of the constant-money-income demand curve has been a mistake.

Friedman's argument in support of the constant-real-income demand curve is based on the verification that the use of an ordinary demand curve in a supply-demand diagram when we analyze the effects of a subsidy in a given commodity fails to take account of the necessary withdrawal of resources from other uses through a corresponding taxation.<sup>1</sup> He argues that compensated demand curves, which in the limit are an approximation to what the community can actually have, allow for this withdrawal of resources, and therefore present a better picture of the final outcome.<sup>2</sup>

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<sup>1</sup>Note that in Friedman's argumentation the unit of decision is not the individual but the community, since he is dealing with the impacts of public policies on the whole economy. The community will be the unit of decision in Bailey's proposal, which will follow.

<sup>2</sup>Friedman, "The Marshallian Demand Curve," pp. 467-474. The method of analysis that we are going to propose

However, it is important to observe that in the analysis of such policies of subsidy and taxation it is not necessarily true that if subsidy outlays and tax receipts are equal, the new line of attainable combinations (ZW in Figure 6A) will be tangent to the initial indifference curve,  $U_2$ . Or, conversely, that if the new line of attainable combinations is tangent to the initial indifference curve, we will have a balance between subsidy outlays and tax receipts.

Suppose that the community faces initially a budget constraint, which corresponds to line MN in Figure 6A, in the form:

$$(MN) \quad Y = P_1 x_1 + P_2 x_2 ,$$

$$\text{or} \quad x_2 = \frac{Y}{P_2} - \frac{P_1}{P_2} x_1$$

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in Part II of this study is an alternative way for estimating the effects of taxation and subsidy on aggregate demand curves.

If the government imposes a subsidy on consumption of  $X_1$ , and its market price is reduced by the unitary amount of this subsidy (which we may call  $s$ ), then the new budget constraint will correspond to the equation:

$$(MQ) \quad Y = (1 - s)P_1x_1 + P_2x_2 ,$$

or

$$x_2 = \frac{Y}{P_2} - (1 - s)\frac{P_1}{P_2} x_1$$

where total subsidy outlays equal  $sP_1x_1$ .

However, if the government collects taxes in a total of  $Y_0$ , reducing the available income by the same amount, then this community will have a second new line of attainable combinations ZW, which will correspond to the equation:

$$(ZW) \quad Y - Y_0 = (1 - s)P_1x_1 + P_2x_2 ,$$

or

$$x_2 = \frac{Y}{P_2} - \frac{Y_0}{P_2} - (1 - s)\frac{P_1}{P_2} x_1$$



What we are alluding to is that even if  $Y_0 = sP_1x_1$ , that is, subsidy outlays equal tax receipts, there is no guarantee that the simultaneous reductions in the slope and intercept of the budget constraint equation will maintain it still tangent to the initial indifference curve,  $U_2$ . To the extent that  $s$ ,  $P_1$ ,  $x_1$ , and  $Y_0$  are relatively large with respect to  $(1 - s)$ ,  $P_2$ ,  $x_2$ , and  $Y$  respectively, and depending on the curvature of the indifference curve  $U_2$ , it will be more likely that such a program will result in a net loss of welfare, and vice versa.

Also, any administrative costs involved with the implementation of these programs would certainly add more chances to the possibility of a net loss of welfare.

That explains why Friedman was very cautious in his statement about compensated demand curves being in the limit an approximation to what the community can actually have. At any rate, Friedman enhanced Demand Theory dramatically by proposing an specific application



for compensated demand curves.<sup>1</sup>

### The Production-Frontier Demand Curve

Bailey<sup>2</sup> published in 1954 a very interesting paper which argued that although Friedman<sup>3</sup> was correct in saying that the classical demand curve should not be used in investigations of the impacts of subsidy and taxation on quantity demanded, he failed to present the best alternative definition for it. In Bailey's words:

. . . I shall contend that Friedman did not make the best choice of a curve as an improvement on the conventional one and that the constant-real-income curve, strictly interpreted, does not on balance possess the superiority he claims for it.<sup>4</sup>

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<sup>1</sup>For further discussions on compensated demand curves see Baumol, Economic Theory, p. 213.

<sup>2</sup>M. J. Bailey, "The Marshallian Demand Curve," Journal of Political Economy 62(1954):255-61.

<sup>3</sup>Friedman, "The Marshallian Demand Curve."

<sup>4</sup>Bailey, "The Marshallian Demand Curve," p. 255.

Bailey defines his production-frontier demand curve in the framework of a closed economy, where the community consumes what it produces in each period, assuming that there is no trade and that there is no storage of commodities from one period to another.

In deriving the production-frontier demand curve, we will allow for trade to occur, so that the community may consume in each period a level different from what it produces. Figure 7 illustrates this derivation. It requires that we add to our conventional graph a production possibilities frontier (HI) which is tangent to one of the indifference curves and the line of attainable combinations (MN) at the initial utility maximizing equilibrium point (A).

Again, we suppose that there is an exogenous decrease in the price of commodity  $X_1$  due to a certain subsidy paid by the government on consumption of  $X_1$ , which shifts the line of attainable combinations from MN to MQ, other things constant. Given this apparent opportunity, the community will prefer to consume at point C,

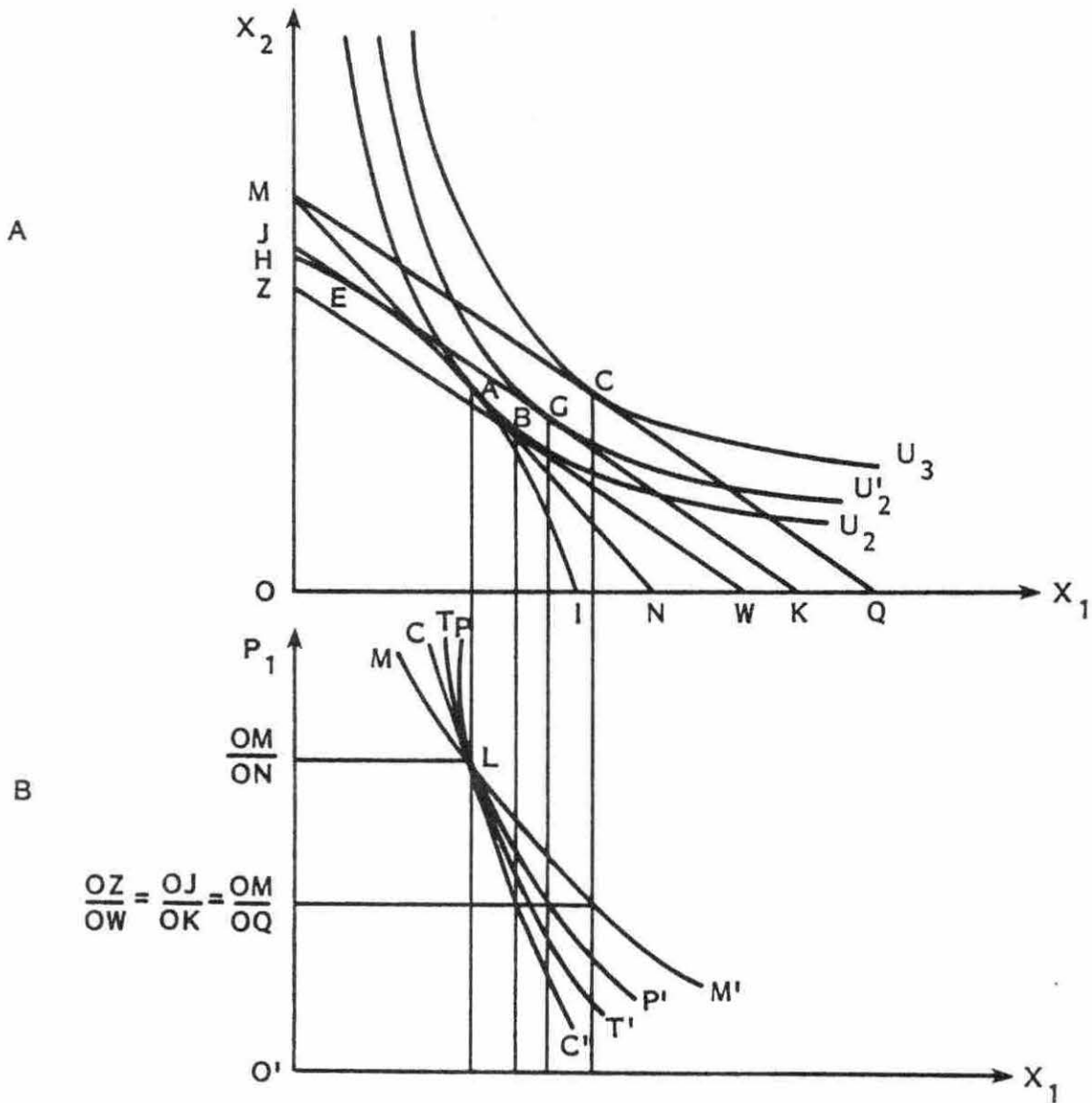


Figure 7. Derivation of the production-frontier demand curve

in Figure 7A. However, as Friedman pointed out, this is clearly impossible since physical supplies are not available and we have to allow either for an inflationary gap, or, instead, suppose that the subsidy is financed by some policy of taxation.

We will assume that the government will tax income at a sufficient rate, such that the marginal rate of transformation equals relative prices (point E). However, although producing at point E, the community will prefer to consume at point G, where the marginal rate of substitution in consumption equals relative prices. This new consumption point is only slightly distant from B, the point at which ZW is tangent to  $U_2$ , which corresponds to Friedman's compensated demand curve.

Bailey argues that it can be seen from this result that the constant-real-income demand curve (CC' in Figure 7B) does not show the final outcome correctly.<sup>1</sup> The correct outcome can be obtained only by the

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<sup>1</sup>Bailey, "The Marshallian Demand Curve," p. 256.

production-frontier demand curve (PP' in Figure 7B), since the final solution must be on the production possibilities frontier.

We conclude stressing that Bailey's proposal is superior to that of Friedman only when the production function is a constraining factor.

Figure 7 illustrates simultaneously the four definitions of demand curves that we reviewed briefly in this chapter. We remember that the curves labeled MM' and TT' are the Marshallian (classical) and the trader's demand curves, respectively.

Finally, the main lesson from this chapter is that there is no such thing as "a demand curve." An economic problem that calls for the use of a demand curve will in general contain the information necessary for deciding which definition of demand curve is relevant to it.<sup>1</sup>

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<sup>1</sup>Dan Usher, "The Derivation of Demand Curves from Indifference Curves," Oxford Economic Papers 17(1965):24-46.



## CHAPTER III.

AGGREGATE DEMAND CURVES  
AND INCOME DISTRIBUTION

This chapter is concerned with the motive for the inclusion of the income distribution factor as a variable in aggregate demand functions. It will focus also on works of the few authors who either limited themselves to recognize the importance of income distribution as a variable, or went further and built models actually including it.

## Aggregation of Demand Curves

Text definitions of demand curves state that they are relations between price and quantity demanded, other things constant, and most of the times refer to the case when the unit of decision is an individual, a household, or a family, but seldom a community. Why is it so more common and simpler to refer to individual demand curves?



For two things, one, because we can easily abstract on a set of indifference curves for the individual containing all the needed properties for derivation of demand curves, whereas to think about indifference curves for a community, we have to make restrictive assumptions to maintain the same properties;<sup>1</sup> two, because even if we assume constant distribution of money income, we have to interpret it in a dynamic manner since any change in the relative prices of commodities would also mean a change in the income held by individuals possessing different bundles of goods, something that certainly complicates the analysis.

Therefore, since the community indifference map changes the distribution of income, the derivation of non-intersecting community indifference curves is not independent from the distribution of income.

An aggregate demand curve suffers basically from the same defects of a community indifference map. When

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<sup>1</sup>Namely, we must suppose that distribution of income is constant, something which we are not interested in, here.

we draw an aggregate demand curve, of course it is the horizontal summation of the individual demand curves. But to know what the aggregate demand is, we not only need to know what is the aggregate income, but also how it is distributed among the decision units. So, in general, just like a change in the distribution of income will shift the community indifference curves, it will also shift the aggregate demand curve, which leads us to conclude that neither of them exist in the sense that they are independent of the distribution of income.<sup>1</sup> We recall Nystrom:

From the foregoing statements it will thus be seen that variations in income of the people of the country constitute the most fundamental factor in consumer demand. A knowledge of income, its distribution and the changes in income trends are obviously of utmost interest <sup>2</sup> to an understanding of the Economics of Consumption.

But then, when will aggregate demand curves be independent from the distribution of income? This will

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<sup>1</sup>We are grateful to the lectures of Professor Harvey Lapan during the Fall of 1980, at Iowa State University.

<sup>2</sup>Paul H. Nystrom, Economic Principles of Consumption (New York: Ronald Press, 1929), p. 158.

happen when: 1) consumers have identical preferences (or identical demand curves); and, 2) their preferences are always homothetic, that is, have unitary income elasticity of demand at all price levels. Relaxing the second condition above we can analyze how income redistribution will affect sectoral aggregate demand curves.

The authorship on this subject may be classified in three distinct groups. First, those authors who do not identify income distribution as a variable in the aggregate demand function, who constitute the great majority, and for whom we will not give any attention. Second, those authors who acknowledge the role of income distribution in aggregate demand functions, but do not go further than that. And third, those authors who incorporate the variable in their models.

### Recognizing the Importance of Income Distribution

Many economists have not even recognized the role of income distribution in aggregate demand functions.

Often they have avoided introducing the concept of income distribution into their rigorous analyses of Utility and Demand Theory believing that statements on such matters imply value judgements or are subject of ethics or morals.<sup>1</sup>

Nonetheless, a few authors like Green<sup>2</sup> recognize the relevancy of considering the distribution of income among consumers for the measurement of aggregate demand functions, albeit they do not develop the idea any further. Some prefer instead to assume that all individual budgets are equal. Indeed, the first one to use this artifice was Marshall, from whom we quote:

The total demand in the place for, say, tea, is the sum of the demands of all the individuals there. Some will be richer and some poorer than the individual consumer whose demand we have just written down; some will have a greater and others will have a smaller liking for tea than he has. Let us suppose that there are in the place a million purchasers of tea, and that their average consumption is equal to his at

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<sup>1</sup>See Jan Tinbergen, An Interdisciplinary Approach to the Measurement of Utility or Welfare (Dublin: The Economic and Social Research Institute, 1972).

<sup>2</sup>H. A. J. Green, Consumer Theory (New York: Academic Press, 1978), pp. 140-2.



each several price. Then the demand of that place is represented by the same list of prices as before, if we write a million pounds of tea instead of one pound.

Others who have only recognized the importance of income distribution on aggregate demand curves include Marschak,<sup>2</sup> De Wolff,<sup>3</sup> and Houthakker.<sup>4</sup>

De Wolff, specifically, recognizes that in general it is not possible to study the relation between aggregate consumption expenditure (C) for a certain commodity in a country and total income (Y) in the country without making some assumption about the character of the income distribution. He points out the special case in which we have expenditure as some linear function of income for all individuals  $j$  in the economy, that is:

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<sup>1</sup>Marshall, Principles of Economics, p. 99.

<sup>2</sup>J. Marschak, "Personal and Collective Budget Functions," Review of Economics and Statistics 21(1939):161-170.

<sup>3</sup>P. De Wolff, "Income Elasticity of Demand: A Micro-economic and a Macroeconomic Interpretation," Economic Journal 51(1941):140-5.

<sup>4</sup>Houthakker, "The Present State of Consumption Theory."

$$c_j = c(y_j) = a + by_j$$

for a and b constants.

In this case, aggregate consumption expenditures would reduce to:

$$C = an + bY$$

where n is the number of decision units, and

$$Y = \sum_{j=1}^n y_j$$

De Wolff<sup>1</sup> concludes that in all other cases we must know the properties of  $F(i)$  -- the income distribution factor -- in order to be able to perform the transition from income elasticity in the microeconomic sense to income elasticity in the macroeconomic sense.

Houthakker in his turn admits that:

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<sup>1</sup>De Wolff, "Income Elasticity of Demand: A Microeconomic and a Macroeconomic Interpretation," p. 141; in Chapter V we will verify that this case corresponds to a linear relation between permanent income and permanent consumption, and that although income redistribution will not affect aggregate consumption, it may still cause shifts in the sectoral aggregate demand curves.



. . . the discussion of market demand in Hicks (1939, esp. para. 12 of the mathematical appendix) may suggest to the unwary that microeconomic theorems can be immediately generalized to aggregates, but in fact this is possible only for severely circumscribed distributions of income.<sup>1</sup>

However, immediately after this passage, he disappoints us saying that ". . . on the other hand, the influence of the income distribution may well be small in reality, especially since this distribution seems to be governed by well-defined if little-understood empirical laws."<sup>2</sup>

The arguments used to justify the non-consideration of income distribution are various, but none is strong. If all individuals have the same indifference map, and have unitary income elasticities of demand at all price levels, then it does not matter which income goes to whom, and we need to impose no restrictions on the distribution of income.

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<sup>1</sup>Houthakker, "The Present State of Consumption Theory," p. 732.

<sup>2</sup>Ibid., p. 732.

To allow for effects of income redistribution we need to relax only the second condition above, of unitary income elasticities of demand at all prices. Allowing for individuals to have different preferences, as Farrell<sup>1</sup> suggests, makes any analysis infeasible. However, if we assume that individuals change preferences with income, and that all individuals have the same preferences at each level of income, then we have the sufficient conditions to analyze and explain the effects of income redistribution on aggregate demand.

#### Empirical Studies Involving Income Redistribution

Among those who built in income distribution as a variable of aggregate demand functions we find first those who dealt with family budget studies. Among them

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<sup>1</sup>M. J. Farrell, "Some Aggregation Problems in Demand Analysis," Review of Economic Studies 21(1953-4): 193-203.

we give regard to Nystrom,<sup>1</sup> who was interested in showing that each level of income, or family income, is a strong variable to measure level of education, health, and standard of living in general. Much later, Chiswick<sup>2</sup> developed this same idea allowing for a location effect.

We also consider of value the work of Canoyer and Vaile,<sup>3</sup> who concluded that when the incomes of specific families change, their consumption patterns also change. More specifically, they present a comparison of consumer expenditure data for two widely separated periods -- 1935-6 and 1948 -- which show differences in the percentage distribution of expenditures. They reckoned that, in general, the proportion of consumer expenditures going for food decreased, for clothing remained about the same, and recreation increased, while income at all levels rose

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<sup>1</sup>Paul H. Nystrom, Economic Principles of Consumption.

<sup>2</sup>Barry R. Chiswick, Income Inequality: Regional Analyses within a Human Capital Framework (New York: Columbia University Press, 1974).

<sup>3</sup>Helen G. Canoyer and Roland S. Vaile, Economics of Income and Consumption (New York: Ronald Press, 1951), pp. 136-40.

approximately two and a half times. These results agree with the trends pointed out by Engel almost one hundred years before.<sup>1</sup>

Prais and Houthakker,<sup>2</sup> under the guidance of and in close association with Stone's earlier work,<sup>3</sup> drew attention on the evidence for a non-linear relation between total expenditure and the expenditure on a particular item, and a tendency towards a saturation level in certain commodities. This evidence was confirmed in later studies such as the one by Liviatan,<sup>4</sup> who found the semi-log formulation for his Engel curves most satisfactory, and by Jorgensen,<sup>5</sup> who suggested that his

<sup>1</sup>Canoy and Vaile, Economics of Income, p. 137.

<sup>2</sup>S. J. Prais and H. S. Houthakker, The Analysis of Family Budgets (London: Cambridge University Press, 1955).

<sup>3</sup>J. R. N. Stone, et al., The Measurement of Consumers' Expenditure and Behaviour in the United Kingdom, 1920-1931 vol. 1 (London: Cambridge University Press, 1954).

<sup>4</sup>N. Liviatan, Consumption Pattern in Israel (Jerusalem: Falk, 1964).

<sup>5</sup>Erling Jorgensen, Income-Expenditure Relations of Danish Wage and Salary Earners (Copenhagen: Denmark Statistical Department, 1965).



Danish data could be better described by a double-log function.

Houthakker and Taylor<sup>1</sup> also found evidence of non-linear Engel curves using data for the United States.

These works, however, only validate something that was already expected, namely that patterns of consumption are a function of income, which only indirectly suggests the existence of effects of income redistribution on aggregate demand.

Budd and Whiteman,<sup>2</sup> Tinbergen,<sup>3</sup> and Pryor<sup>4</sup> realized some other unique attempts of introducing income

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<sup>1</sup>H. S. Houthakker and Lester D. Taylor, Consumer Demand in the United States: Analysis and Projections (Cambridge: Harvard University Press, 1970).

<sup>2</sup>Edward C. Budd and T. C. Whiteman, "Macroeconomic fluctuations and the size distribution of income and earnings in the United States," in Income Distribution and Economic Inequality, eds. Zvi Griliches et alii (Frankfurt: Campus Verlag, 1978).

<sup>3</sup>Tinbergen, Measurement of Utility; Jan Tinbergen, "A Positive and a Normative Theory of Income Distribution," Review of Income and Wealth 16(1970):221-34.

<sup>4</sup>F. Pryor, "Simulation of the Impact of Social and Economic Institutions on the Size Distribution of Income and Wealth," American Economic Review 63(1973):50-72.

distribution into the scene.

Budd and Whiteman constructed a simulation model of the effects of changes in unemployment on the size distribution of income, ranking income from labor by size for adult males and females, and also for households. Their main conclusion is that for the United States, during the period between the '40s and the '70s, increased unemployment resulted in greater inequality in the distribution of income and earnings, but that these distributive effects were small.

Tinbergen in turn, sets the income distribution scale as the unknown variable of his formulation, such that there must be an equalization of supply and demand in all sectors, in what he called his positive approach, and be such that social welfare is maximized, under his normative approach.<sup>1</sup>

Pryor simulates the distribution of income in a multi-generational context. He specifies an "intergenera-

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<sup>1</sup>Tinbergen, "A Positive and a Normative Theory of Income Distribution," p. 222.



tional saving function" which relates bequests to lifetime resources, and allows for two different functional forms. One function assumes that the elasticity of bequests with respect to resources is unity, and the other assumes that bequests are luxury goods, having an elasticity in excess of unity. His results show that the second function yields a substantially greater degree of income inequality than the first function, since as individuals acquire more wealth they will tend to have proportionately greater bequests. These conclusions were later confirmed by Menchik and David.<sup>1</sup>

All these works, however, give little if any attention to the specific matter of the effects of the variable income distribution in patterns of demand. The only empirical work focusing on this very issue has been done by Cline. His work has been the most ambitious,

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<sup>1</sup>Paul Menchik and Martin David, "The Effects of Income Distribution and Redistribution on Lifetime Saving and Bequests," in National Bureau of Economic Research, Conference on the Taxation of Capital (November, n.d.).

if not the only project undertaken in the empirical estimation of the effects of income redistribution on macroeconomic variables.<sup>1</sup>

We shall contend, however, that there is an alternative approach to the study of the effects of income redistribution on demand curves which is superior to the one utilized by Cline.

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<sup>1</sup>William R. Cline, Potential Effects of Income Redistribution on Economic Growth: Latin American Cases (New York: Praeger, 1972).

PART II.

THE EFFECTS OF INCOME REDISTRIBUTION  
ON SECTORAL AGGREGATE DEMAND CURVES

## CHAPTER IV.

## PRELIMINARIES

The central concern of this study is the formulation of a method for analyzing the effects of income redistribution on sectoral aggregate demand curves, as an alternate to the one proposed by William Cline.<sup>1</sup> The literature in this precise matter is very limited, and Cline's work has been the only one to deal with the specific formulation of a methodology of study and to undergo empirical simulation using real data.

We shall contend that our method is superior to that of Cline.

## William Cline

Cline's major concern is to determine the effect which income redistribution could have on economic growth.

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<sup>1</sup>Cline, Potential Effects of Income Redistribution on Economic Growth.

He argues that the effect of income equalization on savings and capital formation is one, and perhaps the major, element in the relationship between equity and growth.<sup>1</sup> Because of that, he devotes considerable importance to the effects of income redistribution on aggregate savings alleging that, in the aggregate, income redistribution will hinder savings, and consequently will slow down the rate of growth of the economy. In this sense, Cline seeks to find among the four major theories of the consumption function<sup>2</sup> one which unambiguously supports his reasoning.

He finds theoretical support for a decline in aggregate savings as income is redistributed from high-

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<sup>1</sup>Ibid., p. 13.

<sup>2</sup>The four alternative hypothesis are: 1) the average propensity to save rises as income rises (Keynesian consumption function); 2) consumption is a constant fraction of permanent income (Friedman); 3) the savings rate is a function of income level relative to average income in the society (Duesenberry); 4) saving is done for the purpose of retirement plus desired bequests, and the savings rate depends mainly on the individual's age (Modigliani and Brumberg).



income to low-income recipients in the curvilinear version of the Keynesian function, which assumes decreasing marginal propensity to consume. Cline rejects the three other theories saying that:

The "permanent income" hypothesis implies no change in savings; the life-cycle hypothesis implies decreased savings only if bequests rise more than proportionately with income; and the relative income hypothesis (like the Keynesian hypothesis) gives decreased savings for some specifications of the function but not for a linear specification.<sup>1</sup>

It should be clear that Cline chose a specification of the consumption function to suit his a priori condition that income redistribution from high-income to low-income recipients will decrease the aggregate level of savings. But we will see ahead that this is not a unique property of the curvilinear version of the Keynesian function.

Whilst Cline's major concern is the impact of income redistribution on economic growth, our major concern is with the method he uses to analyze the effects

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<sup>1</sup>Cline, Potential Effects of Income Redistribution on Economic Growth, p. 19.



of income redistribution on the composition of demand. Cline states that when income is redistributed,<sup>1</sup> the composition of demand shifts away from income-elastic goods and toward income-inelastic goods, something that can be agreed upon only if we make some restrictions on movements of prices and the shape of aggregate supply curves.

To prove his point, Cline presents the example of a two-household economy, where  $y$  is transferred from the high-income household ( $r$ ) to the low-income household ( $p$ ). Figure 8 reproduces Cline's figure in that respect, where  $C$  is consumption expenditure for each good  $i$  as indicated (or  $P_i x_i$ , for  $i = A, B$ ), and  $Y$  is total expenditure in each household ( $r$  or  $p$ ).<sup>2</sup>

We can see that the decline in  $r$ 's expenditure for

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<sup>1</sup>When not specified to the contrary, we will be referring to the case of equalization of income distribution.

<sup>2</sup>Note that Cline refers to part of total expenditure being transferred between the two groups, and not income. In this sense, the problem of consumption decision -- or how much is saved out of income -- is being isolated from this analysis.

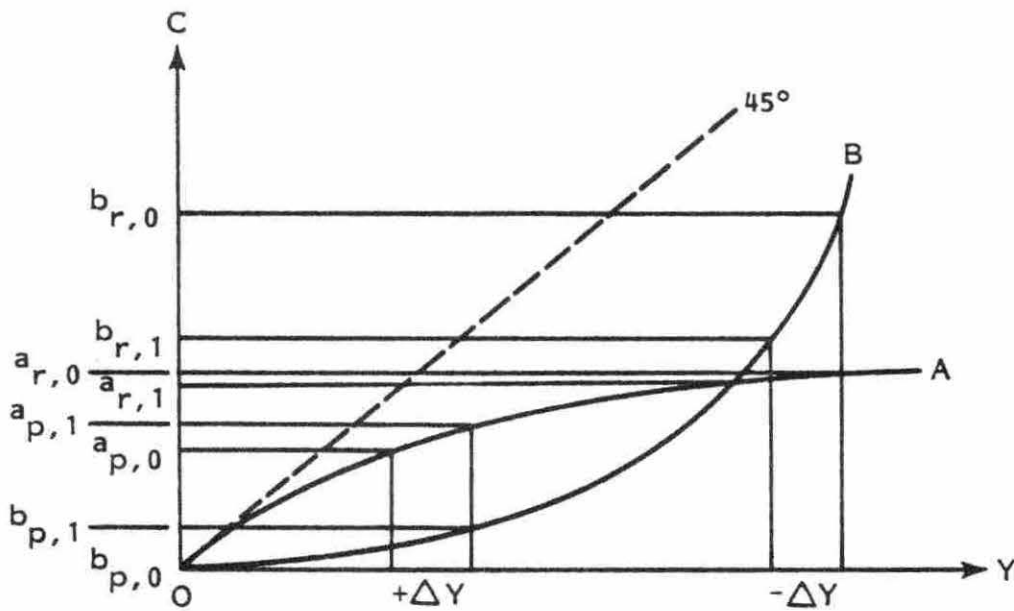


Figure 8. Change in demand for income-elastic and income-inelastic goods following income redistribution

the income-elastic good (B) exceeds the increase of p's expenditure for the good, that is,

$$b_{r,o} - b_{r,l} > b_{p,l} - b_{p,o}$$

Similarly, the increase of p's consumption of the income-inelastic good (A) rises more than r's decline in the consumption of the good, that is,

$$a_{p,l} - a_{p,o} > a_{r,o} - a_{r,l}$$

so that the composition of demand shifts in favor of the income-inelastic good (A).

Cline's empirical estimations of the effects of income redistribution on the composition of demand in Brazil and México utilizes essentially this same procedure, only that it introduces afterwards the individual's decision on how much to save and how much to consume at each level of income.

At any rate, with a small modification, we can show that the curves presented in Figure 8 have a lot in

common with Engel curves, since they are relations between levels of income and expenditure for each good holding prices constant.

#### Unveiling Cline's Method

To derive Engel curves from the curves in Figure 8, first, we separate the curves in individual diagrams, so that commodity A is analyzed in Figure 9A, and commodity B in Figure 9B. Now, if we divide the variables of expenditure for each commodity by their quantities demanded ( $x_i$ ).<sup>1</sup> Then if we invert the axes, we are left with the typical presentation of Engel curves. The convex curve corresponds to the income-inelastic good (A), and the concave curve to the income-elastic good (B).

We argue that Cline's method does not yield precise results, since it takes account of changes in

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<sup>1</sup>This is possible because in such curves prices are assumed to be constant.

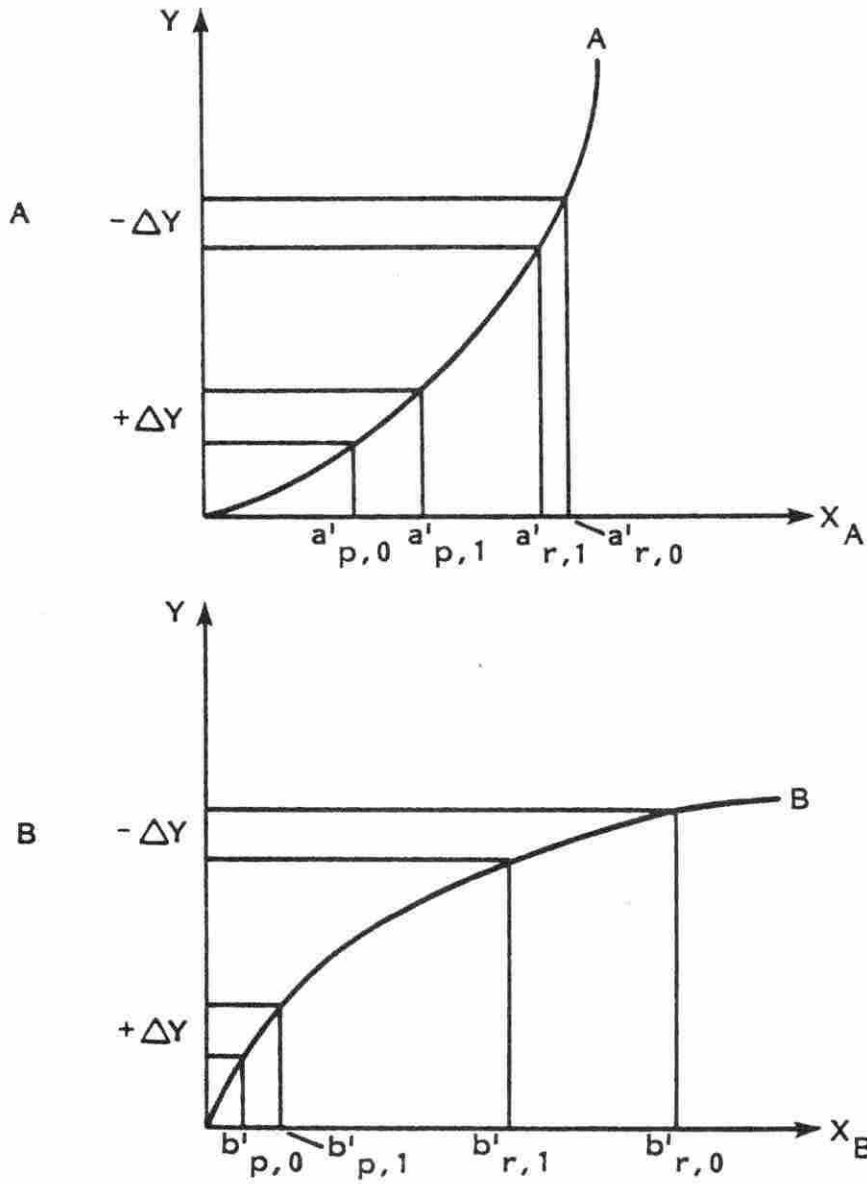


Figure 9. Engel curves for income-inelastic and income-elastic goods



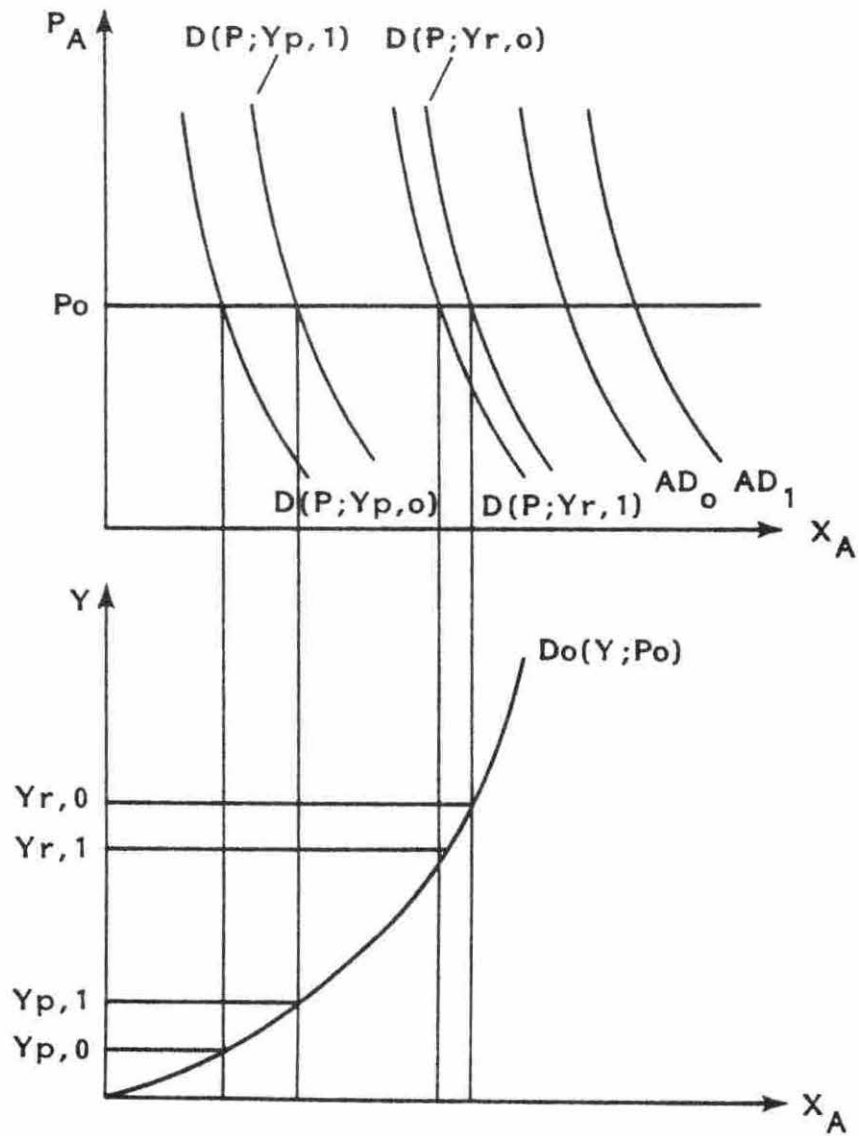
quantity demanded due to income redistribution looking only at Engel curves, implying that relative prices are held constant. But we know that relative prices are not necessarily the same after income redistribution. Changes in the composition of demand will change also the relative prices of goods (and therefore may change their money prices), unless supplies are perfectly elastic.

But then how can we incorporate an eventual change in prices into such analysis? A good start is to investigate how an Engel curve is derived.

Figure 10A illustrates a set of demand curves for good A (which is assumed to be normal) in an expansion which reflects increasing levels of total consumption expenditure (from  $Y_{p,0}$  to  $Y_{r,0}$ ). In this two-household economy, we assume that both households have the same preferences or tastes, meaning that their indifference maps are the same. Thus, at each level of consumption expenditure both will present the same demand behavior.

If A is the income-inelastic good, for higher levels of total expenditure there will be less than

Figure 10. Expansion of demand curves at different levels of consumption expenditure, for an income-inelastic good



proportionate increases in quantity demanded. That is, when expenditure is shifted from the high-income (r) to the low-income household (p), the movement to the right of the demand curve for the low-income one is greater than the movement to the left of the demand curve for the high-income one. Consequently, the aggregate demand curve for this two-household economy, which is the horizontal summation of the individual demand curves, will shift to the right after income redistribution. The effect of income redistribution on quantity demanded measured under Cline's method is given by the distance between curves  $AD_0$  and  $AD_1$ , at the price level  $P_0$ .

That is, Cline estimates the proper shift of aggregate demand curves at the initial equilibrium price levels. However, what is at issue is the new equilibrium values of price and quantity demanded for each good. We contend that Cline's measurement will reflect the true effects only when the aggregate supply curves are perfectly elastic. If they are perfectly inelastic, the result of income redistribution would be only a rise in

the price of goods. And for positively sloped aggregate supply curves, one should expect as a result a rise in price and in the equilibrium quantity consumed of those commodities.<sup>1</sup>

Therefore, any assertion about the impacts of income redistribution on quantity demanded should also include something about the supply side behavior. An analysis like the one held by Cline, which bases its conclusions only on the income elasticity of demand of goods or sectors, implicitly assumes that prices are held constant, and does not allow for the interaction between the demand and supply forces of the economy.

It is our intention that our proposal gives an insight in forecasting the shifts of sectoral aggregate demand curves when income is redistributed. Since we will not be studying the shapes of sectoral aggregate supply curves, it is not our intention to reach final conclusions

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<sup>1</sup>This is consistent with the idea that for basic food items, as an example, which in the short-run have inelastic supplies, income equalization will have little effect on the equilibrium quantity demanded, and a greater impact on prices.

on what should be the new equilibrium levels of quantity demanded, after income redistribution.

Before we proceed, it may be useful to justify which definition of demand curve is going to be used in our proposal.

#### A Choice of Demand Curve

The choice of demand curve reduces to a problem of simply determining what should be held constant in the demand function.

We have seen that classical demand curves reflect that money income is held constant. Compensated demand curves are consistent with real income (utility level) being held constant. Production-frontier demand curves, in turn, assume that the production possibilities frontier is a constraining factor. And trader's demand curves have a greater economic meaning in micro-analyses of individual decision units when we know their good endowments.



Income redistribution refers to the transfer of money income among individuals grouped by income levels. Part of the money income owned by individuals is saved, and the rest of it is spent. The total expenditure of individuals will determine how much they will purchase of each good, at various price levels, establishing a demand schedule for each level of total expenditure. Therefore, in order to analyze how the demand behavior of individuals is affected when income in terms of money is transferred among them, we ought to investigate what are the demand curves at each level of total expenditure.

There is obviously an imbedded component of error in measuring money income of individuals before and after redistribution, since income redistribution is likely to affect relative prices of some commodities, through changes in the composition of demand. Because, in general terms, individuals possess income in the form of commodities -- hours of labor, metals, or rents -- valued at their market prices, their income levels are subject to a change which is indirectly related to the redis-

tribution process. The only way to control this effect would imply knowing the good endowments of each individual. Since this is clearly impossible, we have to rely upon the assumption that there is no significant effect of price changes on the level of income of each individual.

Our working definition of demand curve can be called a "constant-consumer-expenditure demand curve," which resembles the definition of classical demand curve in that it reflects the existence of individual budget constraints in monetary terms. One takes as parameter the total expenditure of each individual, while the other takes as parameter his (her) money income. Separating them we have the individual's consumption decision.

## CHAPTER V.

A METHOD FOR ANALYZING THE  
EFFECTS OF INCOME REDISTRIBUTION  
ON AGGREGATE DEMAND CURVES

The method we are to propose involves at some point regression analyses using information collected through budget studies. In these regressions, the independent variable is the level of consumption expenditure, and the dependent variable is the expenditure for, or quantity demanded of each good or sector in the economy. These regression equations will be nothing else but Engel curves, which will be the essential elements in estimating the shape of the "constant-consumer-expenditure demand curves."

However, not all budget studies present explicitly the information on total consumption expenditure for each income bracket. Some of them, in lieu, present only the value of total income of an individual or family unit. Hence, we need to elaborate some on the

linkage between total income and total consumption expenditure for an individual.<sup>1</sup> This induces us to some reflections on consumption function theories.

### Consumption Function Theories

In general, consumption function theories relate levels of consumption that correspond to different levels of income. However, we need some connection between the level of income and the level of consumption expenditure, and indeed, consumption and consumption expenditure are two distinct concepts. Consumption includes, in addition to purchases of non-durable goods and services, only the use of durables -- measured by depreciation and interest-cost -- rather than expenditures on durables.<sup>2</sup> Consumption

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<sup>1</sup>However, we need to have at least some budget studies presenting data on level of income and level of consumption expenditure, so that we can make some sort of generalization that can be applied to those studies that show data only on income levels.

<sup>2</sup>See William H. Branson, Macroeconomic Theory and Policy (New York: Harper & Row, 1979), pp. 197 and 206.

expenditure, instead, refers to the expenditure on consumer goods in a given period.

In the short-run, the values corresponding to each concept are usually different, unless all purchases of the services of durable goods are in the form of rents, or the economy is in a stationary state where all durable purchases are for replacement.<sup>1</sup> On the other hand, in the medium-run they tend to be equal. In the formulation of the MPS model, for example, it is assumed that after some exogenous tax or subsidy the length of time needed before the new levels of consumption and consumption expenditure stabilize is of approximately three to four years.<sup>2</sup>

Therefore, to eliminate the problem of jumping from consumption to consumption expenditure, we will not assume that all purchases of the services of durable goods are in the form of rents, neither will we assume that the economy is in a stationary state. Instead, we will assume that:

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<sup>1</sup>Ibid., p. 206.

<sup>2</sup>Ibid., pp. 205-6.

### Assumption One

The income redistribution policies that we will be analyzing are not temporary, but reflect permanent decisions; and individuals perceive them as permanent.

This assumption allows us to aver that it is our intent to measure the medium-run (three to four years) effects of income redistribution.

### The Permanent Income Hypothesis

Out of the four major theories of the consumption function,<sup>1</sup> we find the permanent income and the life-cycle hypotheses the ones with greatest appeal and sound reasoning.

We quote Branson in a very important passage:

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<sup>1</sup>They are: the Keynesian consumption function, the relative income, the permanent income, and the life-cycle hypotheses.



Friedman along with Ando-Modigliani, assumes that the consumer (i) wants to smooth his actual income stream into a more or less flat consumption pattern. This gives a level of permanent consumption,  $c_p^i$ , that is proportional to  $y_p^i$  (permanent income):<sup>1</sup>

$$c_p^i = k^i y_p^i$$

. . . if there is no reason to expect these factors ( $k^i$ ) to be associated with the level of income, we can assume that the average  $k^i$  for all income classes will be the same, equal to the population average  $\bar{k}$ . Thus, if we classify a sample of the population by income strata, as is done in the cross-section budget studies, we would expect that the average permanent consumption in each income class i (using subscripts for income classes as opposed to superscripts to denote individuals) would be  $\bar{k}$  times its average permanent income:

$$\bar{c}_{pi} = \bar{k} y_{pi} ,$$

for all income classes i.<sup>1</sup> (Italics mine.)

We would add at this time two observations. We recall that the value of  $k^i$  for a particular individual i is a function of his (her) stage in the life-cycle -- young and retired people will have larger values of  $k^i$

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<sup>1</sup>Ibid., p. 196.

than people in their middle-ages -- and also a function of the ratio of non-human to human wealth and of the income elasticity of bequests. Therefore, the situation when the average  $k^i$ 's for all income classes are the same, and equal to  $\bar{k}$ , reflects one out of many possible behaviors of the relation between permanent income and permanent consumption.

Under Friedman's reasoning,<sup>1</sup> a series of assumptions<sup>2</sup> provide the elements for the explanation of the cross-section result that marginal propensity to consume (MPC) is smaller than average propensity to

<sup>1</sup>Milton Friedman, A Theory of the Consumption Function (Princeton: Princeton University Press, 1957).

<sup>2</sup>The assumptions say that measured income for an individual<sub>i</sub> in a given period is made up of permanent income,  $y^i_p$ , plus a random transitory income component,  $y^i_t$ , which<sup>p</sup> can be positive, negative, or zero. Similarly, measured consumption in any period is permanent consumption,  $c^i_p$ , plus a random transitory consumption component,  $c^i_t$ , which can also be positive, negative, or zero. Furthermore, Friedman assumes that there is no correlation between transitory and permanent incomes; no correlation between transitory consumption and permanent consumption; and no correlation between transitory income and transitory consumption.

consume (APC), even when the basic hypothesis of the theory is that the ratio of permanent consumption to permanent income is a constant  $\bar{k}$ . This cross-section result is what is found in the budget studies which reflect the consumption decisions of individuals, or households, at a certain point in time. It is illustrated in Figure 11 as the dotted line.

The data that we obtain from those budget studies that give information on both the levels of income and consumption expenditure are respectively average measured income ( $\bar{y}_i$ ) and average measured consumption expenditure ( $\bar{c}_i$ ) for each income class  $i$ .<sup>1</sup> Examples of the types of observations that we could acquire from one budget study correspond to the points on or around the dotted line in

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<sup>1</sup>Note that we can plot the information on consumption expenditure in the consumption-income space because we are treating consumption expenditure and consumption as equivalent concepts. We point out that this is not so weak an assumption as it may have looked at first sight because for an average of many individuals, the expenses with durables tend to be very close to the value of consumption corresponding to their depreciation and interest-cost, or at most we assume that this is the case.

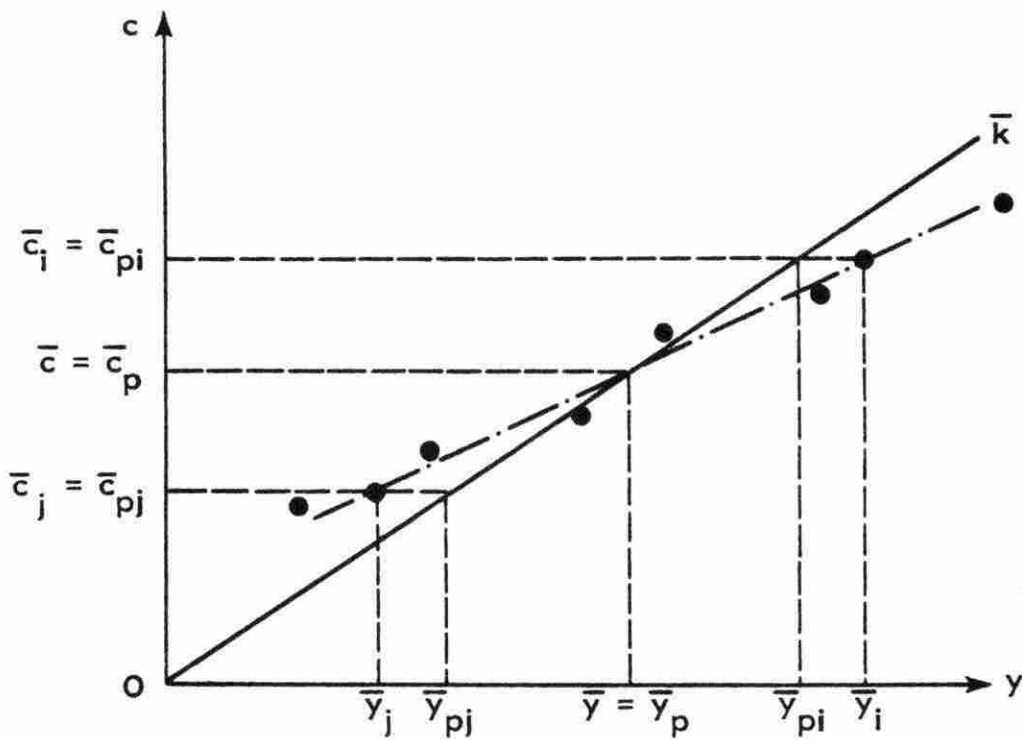


Figure 11. The cross-section consumption curve and the linear relation between permanent income and permanent consumption

Figure 11.<sup>1</sup>

However, since it is assumed that each individual (or household) views the change in his (her) income as being permanent, we need to analyze how income redistribution affects consumption decisions through the relation between permanent income and permanent consumption. And this relation may be linear or non-linear.

Linear relations reflect the case when the average  $k^i$ 's for each income group are the same and equal to  $\bar{k}$ . This will happen when income elasticity of bequests is equal to one, and when the age distribution of individuals and the ratio of non-human to human wealth in each income group do not affect the average  $k^i$ 's for each one (or their effects are cancelled when taken together).

Non-linear relations between permanent income and permanent consumption will come about when these conditions

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<sup>1</sup>Such points plotted in Figure 11 serve only as an illustration, and do not reflect observations from any real data.



are broken down. Menchik and David<sup>1</sup> proved that in the U. S. bequests rise more than proportionately with income.

For analytical purposes, if we look at the case when the relation between permanent income and permanent consumption is non-linear, it will become clear what happens when the relation is a straight line.

Let  $Y$  be the aggregate permanent income for all individuals in the economy, and  $C$  be their total consumption expenditure. Suppose that there are only two income groups: the low-income group ( $p$ ) and the high-income group ( $r$ ), with  $n$  and  $m$  individuals each, respectively. Then, we could write:

$$Y = ny_p + my_r$$

$$C = nc_p + mc_r$$

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<sup>1</sup>Menchik and David, "The Effects of Income Distribution and Redistribution on Lifetime Saving and Bequests."

where  $y_i$  and  $c_i$  are the average income and consumption of individuals in each income group  $i$ , for  $i = p, r$ . Moreover, let us define the relation between permanent income and permanent consumption as:

$$c = f(y; \dots)$$

If we assume that all the income that is taken from individuals in the high-income group is transferred to individuals in the low-income group, then we could write:

$$dY = 0 = ndy_p + mdy_r$$

and since

$$dC = ndc_p + mdc_r$$

and

$$dc = \frac{\partial f}{\partial y} dy$$

then,

$$dC = n \frac{\partial f}{\partial y_p} dy_p + m \frac{\partial f}{\partial y_r} dy_r$$

and substituting in the budget constraint condition,

$$dC = ndy_p \left[ \frac{\partial f}{\partial y_p} - \frac{\partial f}{\partial y_r} \right]$$

We know that  $\underline{n}$  is a positive number, and if income is being transferred from the high-income group to the low-income group, then  $dy_p$  is also positive. Then, the slopes of the relation between permanent income and permanent consumption around the values  $y_p$  and  $y_r$  will determine whether aggregate consumption expenditure would increase or decrease after income redistribution.

If its slope is always decreasing with income, like in Figure 12, then

$$\frac{\partial f}{\partial y_p} > \frac{\partial f}{\partial y_r} ,$$

which corresponds to income elasticity of bequests greater than one, other things equal, and implies that after income redistribution

$$dC > 0$$

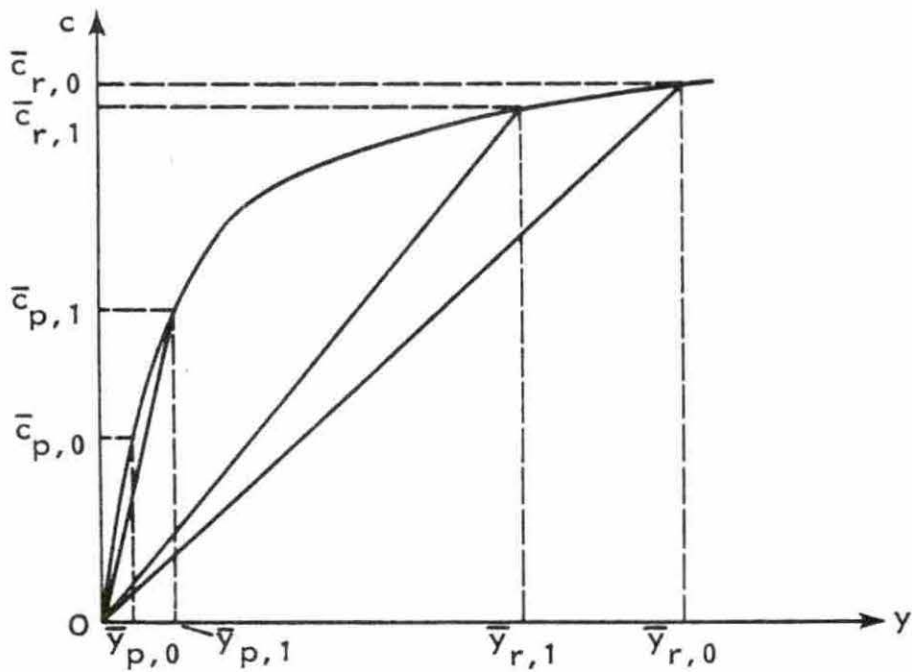


Figure 12. The relation between permanent income and permanent consumption when income elasticity of bequests is greater than one, other things equal

Suppose now that the slope of the permanent consumption-permanent income curve is generally decreasing, but that for an specific range of permanent income it increases with income, such that there is one or more inflection points. Then, there may be a case where

$$\frac{\partial f}{\partial y_p} < \frac{\partial f}{\partial y_r}$$

and income redistribution from the high-income group to the low-income group would decrease aggregate consumption expenditure, increasing aggregate savings.<sup>1</sup>

In the case of a linear relation between permanent income and permanent consumption,<sup>2</sup> we would have

$$\frac{\partial f}{\partial y_p} = \frac{\partial f}{\partial y_r}$$

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<sup>1</sup>This conclusion will be valid, obviously, if the slope of the permanent consumption-permanent income relation is always increasing.

<sup>2</sup>But not only in this case.



and in this instance there will be no change in aggregate consumption due to income redistribution.

Therefore, the impact of income redistribution on the level of aggregate consumption expenditure will be dependent upon the behavior of the relation between permanent income and permanent consumption.

Estimating the Relation between  
Permanent Income and  
Permanent Consumption

The permanent consumption-permanent income curve may be estimated in two different ways. One, we may take various similar<sup>1</sup> budget studies realized in different years, and from each one identify one point in the income-consumption space, which would correspond to the estimated levels of average permanent income and average

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<sup>1</sup>Where similar corresponds to budget studies which are collected in the same manner, with the same precision and consistency, and from the same population.

permanent consumption for the economy. Such collection of points would enable us to estimate the relation between permanent income and permanent consumption.

However, limitations may be found in that a small number of budget studies may be available, and that the methodology used in each one may be significantly different. Note also that this procedure would differ very little, in concept, with a time series analysis of income and consumption.

A second alternate way of estimating the relation between permanent income and permanent consumption is to use directly the data from "controlled" or panel budget studies. In such surveys, each consumer unit in the sample is visited by an interviewer periodically over a certain period of time, who collects data on average income and average consumption expenditure for each consumption item during that interval. It is easy to deduce that if such a survey is carried on over a long period of time, it is likely that the average values of income and consumption expenditure reported for each

family, or individual, will be good estimates of what their permanent income and consumption really are.

The main obstacle to this alternative is that panel budget studies are very expensive, and consequently rare. Also, they may refer to a limited period of analysis. The Bureau of Labor Statistics in the U. S. conducted a panel survey in 1972-1973 in which families reported information to interviewers every 3 months over a 15-month period. In both 1972 and 1973, the sample for the survey was about 10,000 families. The ideal survey for our purposes would be one in which families were asked to report information over a longer period, of 4 or 5 years, for example.

Therefore, the first step in our method is:

#### Step One

Estimate the relation between permanent income and permanent consumption for the economy in focus, using one of the procedures mentioned above.

Of course, if we gather budget studies realized at different periods of time, we must remember to bring all the information from the different budget studies into the same monetary unit. We accomplish this discounting the data utilizing some measure of the general price index, say, the consumer price index ( $P_t$  will stand for the consumer price index in period  $t$ ).

The estimated permanent income-permanent consumption curve may be linear or curvilinear. And we have seen that if it is linear, the aggregate level of consumption expenditure will not change as income is redistributed.

#### Deriving Engel Curves

For the derivation of Engel curves for each sector at various levels of price for the sector, we need to have access to as many budget studies as possible.

Our next step is:

## Step Two

Derive Engel curves for each good or sector in the economy from each budget study separately.

These Engel curves will be relations between consumption expenditure (the independent variable), which is the income available for expenditure after the decision on savings is made, and quantity demanded for each good or sector (the dependent variable).

A few problems in estimating Engel curves may occur.

Possible Problems      The information for the derivation of each Engel curve might not be available directly in some budget studies. There may be budget studies which will present only the average income for each income bracket, not showing how much of that average income was saved.<sup>1</sup> If that is the case, we utilize the

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<sup>1</sup>Note that we are concerned with consumption demand. The demand for investment, which depends on the level of savings, is not considered in this analysis.

permanent income-permanent consumption curve to estimate how much of that total income was actually spent.

Another eventual problem is that the information on quantity demanded of each good or sector will be seldom available in budget studies. Usually, the information that can be found in budget studies is the expenditure, in monetary terms, for each good or sector. If that is the case, we divide the value in monetary terms by a price index for that good or sector ( $P_{it}$  will stand for the price index for good  $i$  in period  $t$ ) for the year corresponding to the budget study ( $t$ ). What we obtain is not quantity demanded, but it is some equivalent measure which can be compared with data from other years derived in the same manner.

A third problem may be the fact that some budget studies present data corresponding to family units instead of individuals. Since in our formulation the unit of decision is the individual, it would be only a matter of dividing the information on average family income and consumption by the average number of individuals in each family group.



Overcoming these possible initial obstacles, we are ready to estimate an Engel curve for each good or sector using data from each budget study at the time. The so-derived Engel curves may be linear, concave from below, or convex from below, corresponding respectively to the cases of unitary income elasticity of demand, income-elastic, and income-inelastic goods.<sup>1</sup>

However, we must be sure that each Engel curve is estimated from observations of only one budget study, since Engel curves are in concept the relation between the level of consumption expenditure and quantity demanded, holding prices constant, and overtime the real price of a good may fluctuate.

We may find whether the price of a certain good varied overtime dividing its own price index ( $P_{it}$ ) by the general price index ( $P_t$ ). And, indeed, in our proposal, in order to derive the demand curves at each level of consumption expenditure, it is essential that prices

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<sup>1</sup>See Appendix II for a review of these relations.

fluctuate overtime.

### Step Three

For each good (or sector), plot in a price-quantity diagram the points corresponding to many levels of consumption expenditure, using each Engel curve derived in Step Two.

Figure 13 illustrates the case of an hypothetical normal good  $n$ .<sup>1</sup> Suppose that using the data from a budget study realized in period 0, we derived an Engel curve for this good  $n$  which is convex from below (Figure 13B). That tells us that good  $n$  at the price level  $P_{no}/P_0$  is income-inelastic.<sup>2</sup>

Step Three tells us to choose arbitrarily which levels of consumption expenditure are we interested in, and to plot in the price-quantity diagram -- Figure 13A -- the corresponding points for those levels of consumption

<sup>1</sup>In Figures 13, 14, 15 and 16 we will be working with examples of normal goods.

<sup>2</sup>Where  $P_{no}$  is the price index for good  $n$  in period 0, and  $P_0$  is the general price index in period 0.

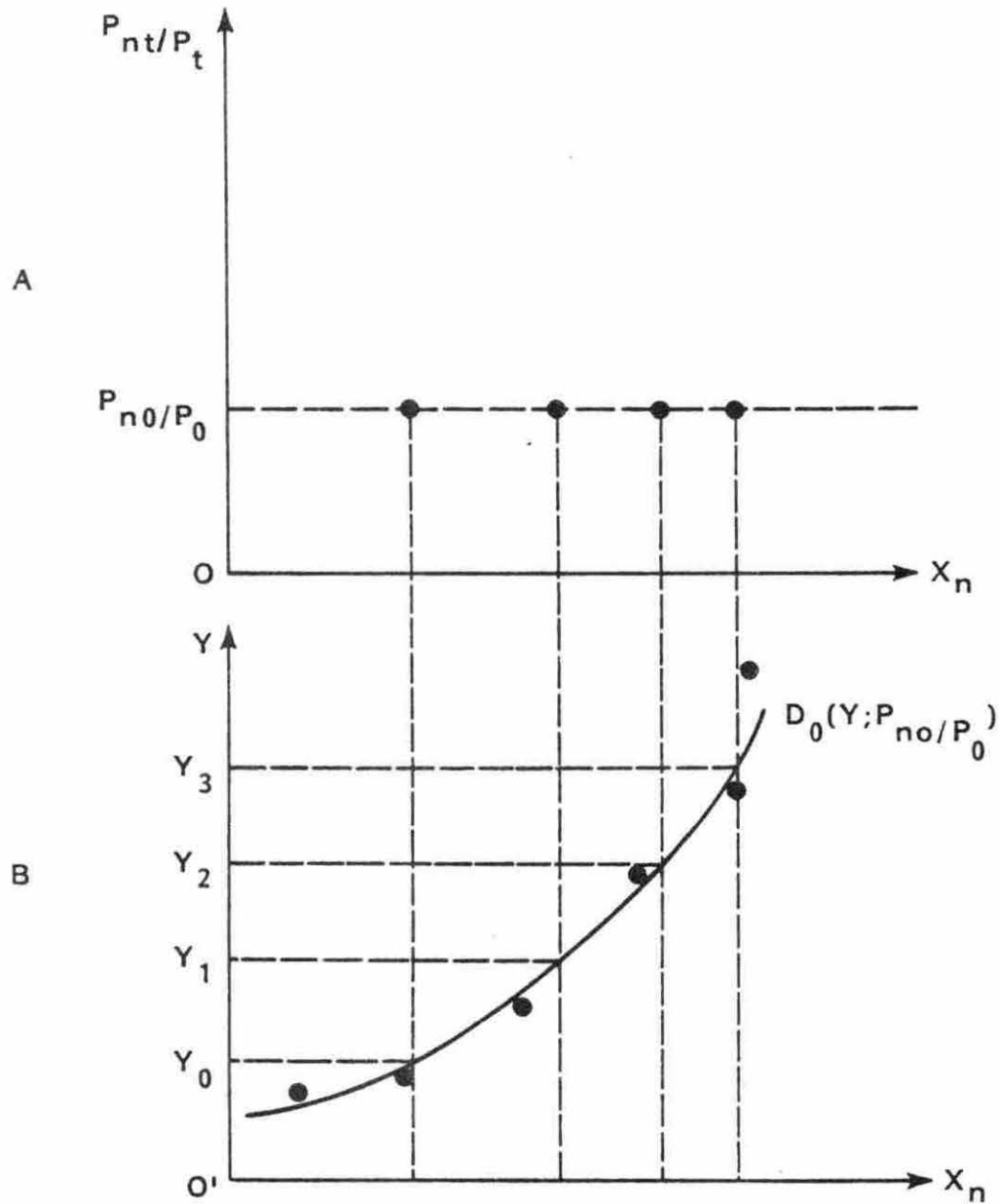


Figure 13. Deriving demand curves for a normal good at each level of consumption expenditure - I

expenditure.

We may repeat this same procedure with an Engel curve derived from another budget study, realized in period 1, which is also convex from below as it is illustrated in Figure 14B.<sup>1</sup> Then, we plot in the price-quantity space (Figure 14A) the points, at price level  $P_{nl}/P_1$ , which correspond to the same levels of consumption expenditure which were arbitrarily chosen previously.

Note that the extent to which  $P_{nl}/P_1$  will be different from  $P_{no}/P_0$  depends on peculiarities of the economy we are analyzing.

#### Deriving "Constant-Consumer- Expenditure Demand Curves"

If we repeat Step Three using Engel curves for many different periods, we will end up with a collection

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<sup>1</sup>Although it is not necessary for it to be so. See in Appendix II the case of goods which can be income-elastic or income-inelastic, depending on the price level.

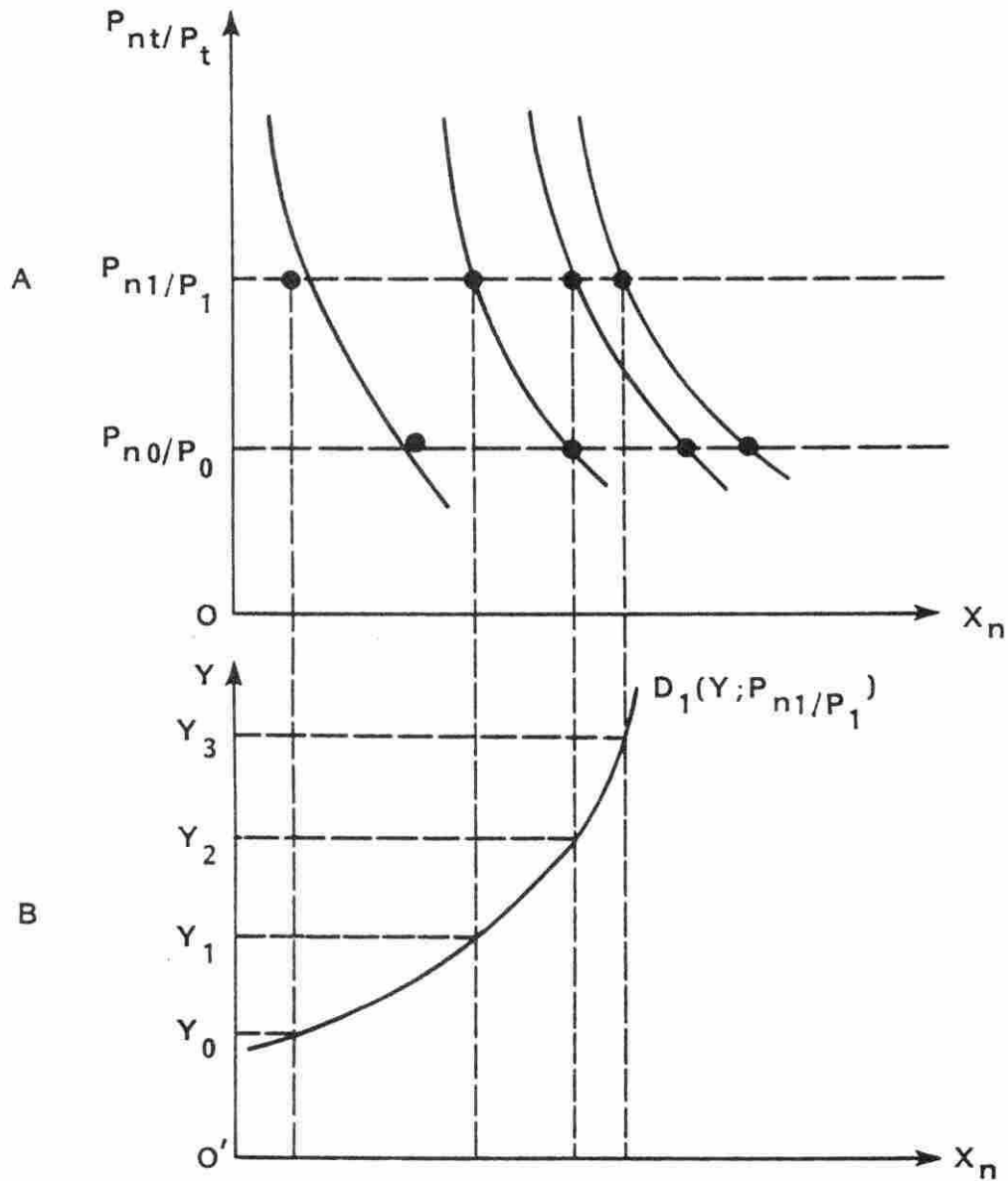


Figure 14. Deriving demand curves for a normal good at each level of consumption expenditure - II

of points in the price-quantity space for each level of consumption expenditure. The next step is:

Step Four

For each good (or sector), using the collection of points in the price-quantity space, estimate for each level of consumption expenditure the respective demand curves.<sup>1</sup>

Figure 14A illustrates the derived "constant-consumer-expenditure demand curves" for good n. Such a diagram enables us to understand how the demand curve for an individual shifts as his level of consumption expenditure changes.

To derive the aggregate demand curve for each good (or sector) in a particular year, we need to collect information on how the national income was distributed among individuals.

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<sup>1</sup>This procedure involves errors in variables, since the variable quantity demanded is not measured without error (its values are derived from the regressions of Engel curves). Also, the fact that we derive continuous relations imply divisibility of goods.



Suppose that we are able to know the number of individuals at each level of average permanent income in that period. Recurring to the relation between permanent income and permanent consumption, we may determine what is the level of average consumption that corresponds to each level of income.

We assume that:

Assumption Two

All individuals have the same preferences and tastes, that is, there is only one indifference map which is common to all individuals.

Assumption Three

The individuals' indifference map does not change overtime. Their income may change, but not their preferences.

The role of Assumption Two is to guarantee that two individuals with the same income will spend it in the same way, which is a basic condition for us to draw any meaningful pattern of consumption behavior.

Assumption Three builds upon the previous one, and essentially says that one individual whose income has increased to  $y_1$ , for example from period 0 to period 1, will have the same preferences and consumption behavior as another individual who had that same level of income,  $y_1$ , in period 0.

Then, we proceed to our next step:

#### Step Five

Add horizontally the demand curves for each good, so that demand curves corresponding to each level of consumption expenditure are added as many times as the number of individuals at each level of consumption expenditure.

Thus, we obtain sectoral aggregate demand curves for a particular period. A good way to check whether our estimation is an approximation to reality is to verify if the share of national income spent in that sector in that particular period falls on the derived sectoral aggregate demand curve.

With these sectoral aggregate demand curves on hand, we may consider two cases for estimating how they would shift if income was redistributed among individuals. First, we will consider the case when the permanent income-permanent consumption curve is linear. And secondly, the case when the permanent income-permanent consumption curve is curvilinear.

#### The Pure Income Elasticity Effect

If the permanent income-permanent consumption curve is linear, transferring income from one income bracket to another will not affect the aggregate level of consumption, provided that all the income that is taken from some income brackets is given to others.

We may simulate redistribution of income assigning redistribution factors to each income bracket.

#### Step Six

Assign income redistribution factors to the average permanent income of individuals in each income

bracket, so that all the value that is taken from some income classes equals the value given to other income classes.

These income redistribution factors must behave as income taxes or subsidies attributed to individuals at each level of income.

Now, we want to investigate what must be the relation between the income redistribution factors if we are to hold the condition that all income taken from one income bracket is given to another one. We will be considering the case of income redistribution between two income groups, only. The high-income group (r) is supposed again to have m individuals, and the low-income group (p) to have n individuals.

Suppose that we know from the permanent income-permanent consumption curve that (remember that this curve is assumed to be linear in this case):

$$c_{r,o} = a + by_{r,o} ,$$

and 
$$c_{p,0} = a + by_{p,0}$$

where  $c_{i,0}$  and  $y_{i,0}$  are, respectively, the average permanent consumption and the average permanent income for individuals in the  $i^{\text{th}}$  income bracket ( $i = p, r$ ), before income redistribution.

If the government imposes an income tax at the rate of  $t$  on individuals in income bracket  $r$ , and an income subsidy at the rate of  $s$  on individuals in income bracket  $p$ , then the new levels of average income for individuals in each income bracket will be:

$$y_{r,1} = (1 - t)y_{r,0} ,$$

and 
$$y_{p,1} = (1 + s)y_{p,0}$$

And the new levels of average consumption expenditure for individuals in each income bracket will be:

$$c_{r,1} = a + b(1 - t)y_{r,0} ,$$

and

$$c_{p,1} = a + b(1 + s)y_{p,0}$$

We know that:

$$m(y_{r,1} - y_{r,0}) + n(y_{p,1} - y_{p,0}) = 0$$

Substituting in the values of  $y_{r,1}$  and  $y_{p,1}$ , we get the relation:

$$\frac{t}{s} = \frac{ny_{p,0}}{my_{r,0}},$$

which tells us what is the condition that has to be satisfied if all the income that is taken from income group  $r$  is given to income group  $p$ , and consequently aggregate consumption is held constant.

In this case, although income redistribution will not change the aggregate level of consumption, it will affect the manner in which the national income is distributed among individuals, that is, the number of



individuals corresponding to each level of consumption, and therefore, the number of times we have to add the demand curves at each level of consumption expenditure.

### Step Seven

Add horizontally the demand curves for each good, so that demand curves corresponding to each level of consumption expenditure are added as many times as the number of individuals, after redistribution of income, at each level of consumption expenditure.<sup>1</sup>

Whether the sectoral aggregate demand curves will shift after income redistribution will depend only on the income elasticity of demand for each good, at each price level, since aggregate consumption is held constant.

We can be certain that the sectoral aggregate demand curves will not shift only in the case of those goods which have unitary income elasticity of demand at

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<sup>1</sup>Note that the levels of consumption expenditure arbitrarily chosen in Step Three for the derivation of the demand curves, should include the new levels of consumption expenditure after income redistribution for the various income groups.

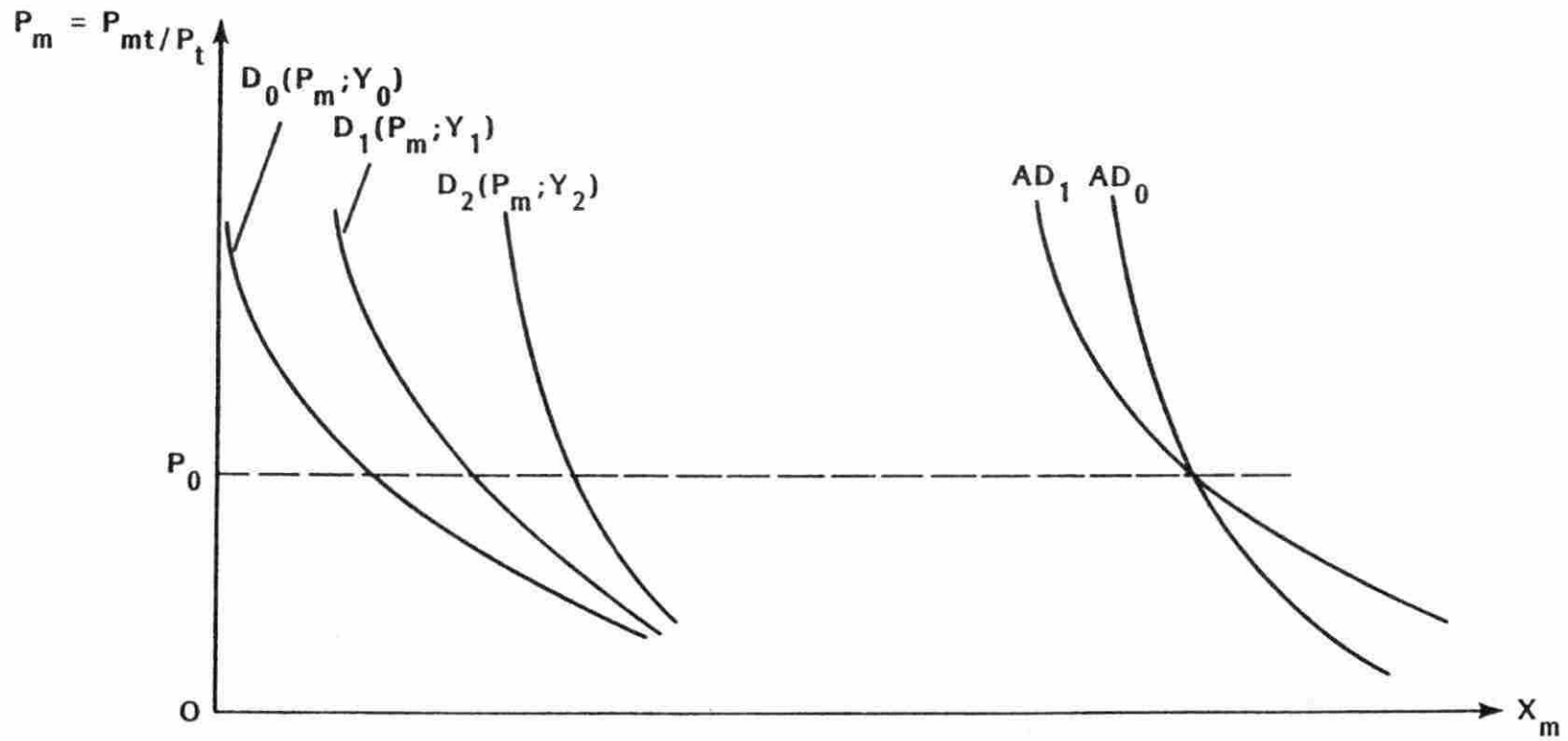
all price levels.

Figure 15 depicts the case of a good  $\underline{m}$ , which has unitary income elasticity of demand at price level  $P_0$ , but is income-elastic at prices above  $P_0$ , and is income-inelastic at prices below  $P_0$ . In this case, if we redistribute income from individuals in higher income brackets to individuals in lower income brackets, then the new aggregate demand curve  $AD_1$  will cross the old one,  $AD_0$ , at price  $P_0$ , and  $AD_1$  will be more elastic than  $AD_0$ .

If we consider another good,  $\underline{i}$ , which is income-inelastic at all price levels, then the redistribution of income from higher to lower income brackets will result in the aggregate demand curve shifting to the right. Figure 16 illustrates this case, where  $AD_0$  and  $AD_1$  are, respectively, the aggregate demand curves before and after redistribution.

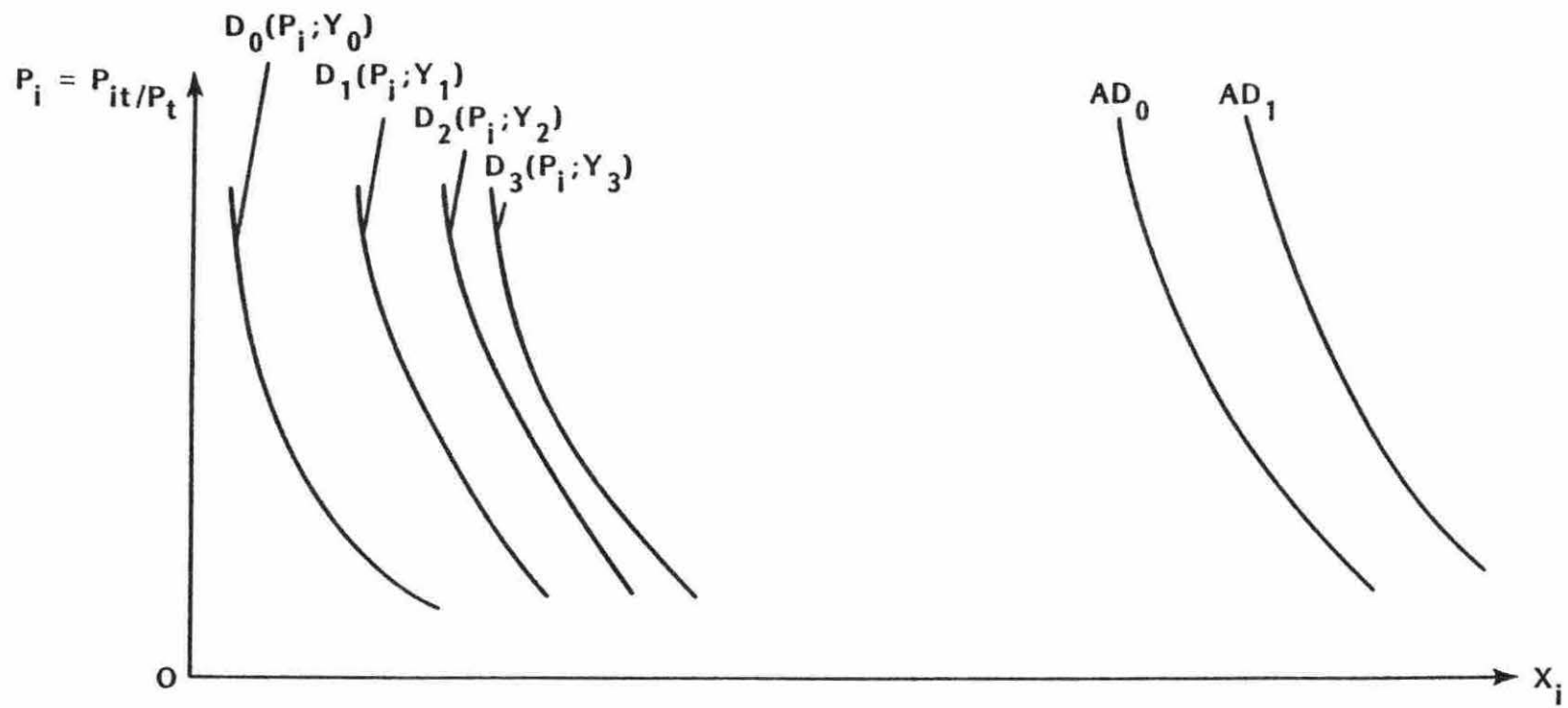
Similarly, if there is a good  $\underline{e}$ , which is income-elastic at all price levels, redistribution of income from higher to lower income brackets will shift the aggregate demand curve to the left. Figure 17 depicts this case.

Figure 15. Effects of income redistribution on aggregate demand curve for a good which is income-elastic and income-inelastic at different price levels



where  $(Y_2 - Y_1) = (Y_1 - Y_0)$

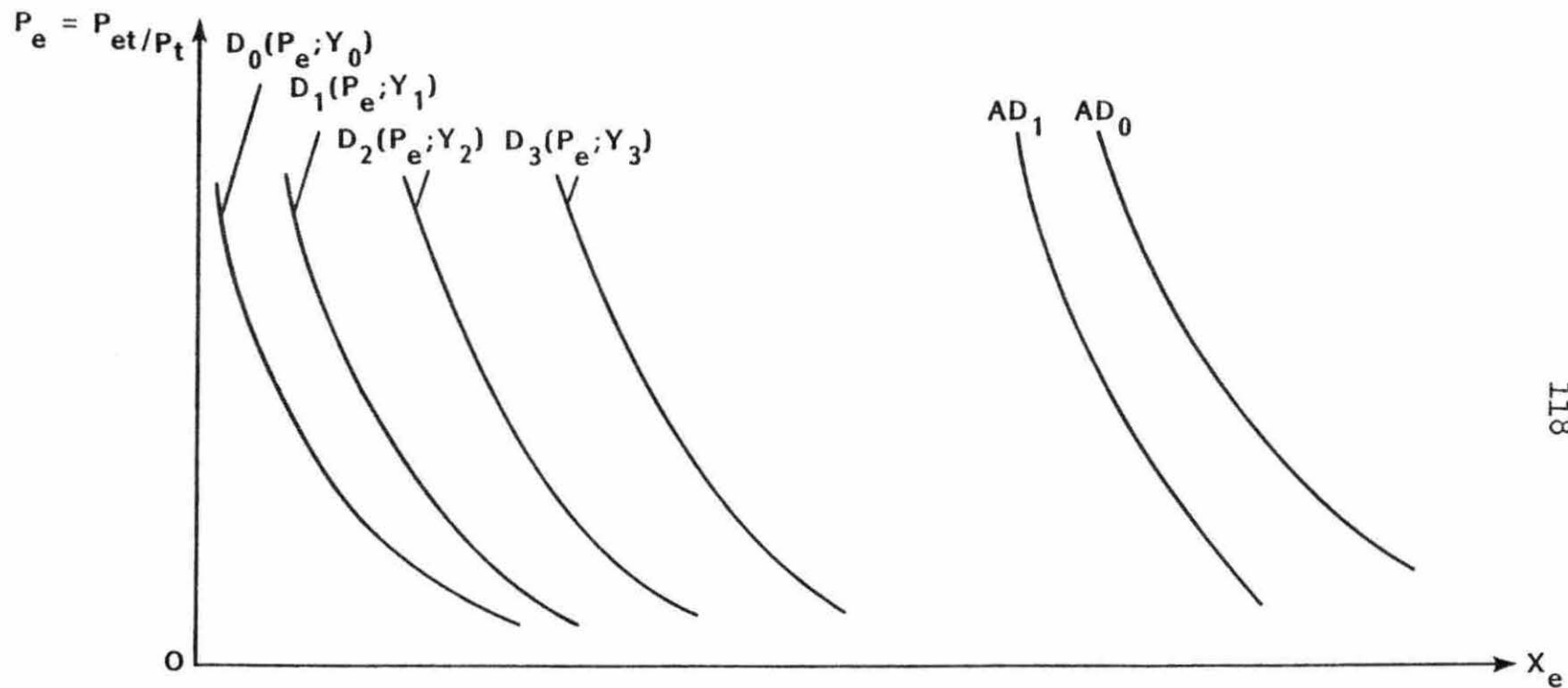
Figure 16. Effects of income redistribution on aggregate demand curve for income-inelastic good at all price levels



where  $(Y_3 - Y_2) = (Y_2 - Y_1) = (Y_1 - Y_0)$



Figure 17. Effects of income redistribution on aggregate demand curve for income-elastic good at all price levels



where  $(Y_3 - Y_2) = (Y_2 - Y_1) = (Y_1 - Y_0)$

Note, however, that these shifts of the aggregate demand curves are occurring even with no change in the aggregate level of consumption, since the permanent income-permanent consumption curve is assumed to be linear in these cases. We call this "the pure income elasticity effect" of income redistribution on aggregate demand curves.

What will happen if we have a curvilinear permanent income-permanent consumption curve?

#### The Aggregate Consumption Effect

As we have already seen, a curvilinear relation between permanent income and permanent consumption is a necessary but not sufficient condition for a change in aggregate consumption after income redistribution. That is, if the permanent income-permanent consumption curve is curvilinear, we may have a change in aggregate consumption.

In the curvilinear case, we should still apply Steps Six and Seven. However, in this case, we know that

when we hold the condition that all income taken from some income brackets is given to others, we can not be sure that aggregate consumption will remain the same. If it remains unchanged, then the only effect that we may have on the sectoral aggregate demand curves is the pure income elasticity effect.

We may investigate under which conditions will aggregate consumption be held constant after income is redistributed between two income brackets, if we have a curvilinear permanent income-permanent consumption curve. We will follow the case where income elasticity of bequests is greater than one, other things equal, which was depicted by Figure 12.

The application of the same income redistribution factors that were used in the previous case imply that:

$$dy_r = (-t)y_r \quad ,$$

and

$$dy_p = (s)y_p$$

If all income taken from group  $r$  is given to group  $p$ , then:

$$m dy_r + n dy_p = 0$$

or the following condition must hold:

$$\frac{ny_p}{my_r} = \frac{t}{s} \quad (1)$$

Suppose that the curvilinear permanent income-permanent consumption curve is in the form:

$$c = f(y; . . .)$$

If we want to know what must be the condition for maintaining aggregate consumption at the same level, after income redistribution in the case of a curvilinear permanent income-permanent consumption curve, then:

$$C = nc_p + mc_r$$

$$dC = 0 = ndc_p + mdc_r$$

And we can find easily that, besides equation (1), the following relation must also hold:

$$\frac{ny_p}{my_r} = \frac{\frac{\partial f}{\partial y_r} t}{\frac{\partial f}{\partial y_p} s} \quad (2)$$

which implies that for this to occur, we must have:

$$\frac{\partial f}{\partial y_r} = \frac{\partial f}{\partial y_p}$$

Now, let us allow for a non-zero effect on aggregate consumption due to income redistribution. Suppose that we have a relation between permanent income



and permanent consumption such that income redistribution from higher to lower income brackets will increase the level of aggregate consumption. In this case, besides the shift that the sectoral aggregate demand curve will suffer due to the pure income elasticity effect, they will shift also due to the increase in the aggregate level of consumption -- "the aggregate consumption effect."

In general, if the aggregate consumption increases after income redistribution and the good is normal at all price levels, then the new aggregate demand curve which accounts for the whole effect will fall to the right of the new aggregate demand curve which would correspond only to the pure income elasticity effect.

Although we are able to identify in concept these two different effects -- the pure income elasticity effect and the aggregate consumption effect -- of income redistribution on sectoral aggregate demand curves, we can not precisely measure each effect separately without compromising with some linear approximation to the relation between permanent income and permanent consumption.

### Final Remarks

This method for analyzing the effects of income redistribution on sectoral aggregate demand curves enables us to determine how these curves will shift after income is redistributed.

Cline's method is only able to identify one point on the new sectoral aggregate demand curves, implicitly restricting its applications and validity to those cases when the sectoral aggregate supply curves are perfectly elastic.

## APPENDIX I.

DEDUCTIONS FROM  
EDGEWORTH'S CONTRIBUTION

As we have pointed out already, Edgeworth's great achievement in Utility Theory was the formulation of the generalized utility function. This Appendix is dedicated to prove how this new specification of the utility function allows for ordinary demand curves with positive slopes and Engel curves with negative slopes.

Stigler<sup>1</sup> was the one who first deduced this possibility by exploring the case of only two commodities. Since investigation of this subject out of the two-commodity world is beyond our scope, we will follow hereafter Stigler's reasoning.

It is crucial to emphasize that, with the additive utility function, diminishing marginal utility for each

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<sup>1</sup>Stigler, "The Development of Utility Theory," p. 323.

commodity was not necessary. For one thing, the indifference curves could still be convex to the origin in the two-commodity case if just one commodity yielded diminishing marginal utility, provided the marginal utility of the other one did not increase too rapidly.

With the generalized utility function, diminishing marginal utility was neither necessary nor sufficient for convex indifference curves. In the two-commodity case:<sup>1</sup>

$$\frac{dX_2}{dX_1} = \frac{-U_1}{U_2}$$

is the slope of an indifference curve, and the condition for convexity is:

$$\frac{d^2 X_2}{d X_1^2} = - \frac{U_2^2 U_{11} - 2U_1 U_2 U_{12} + U_1^2 U_{22}}{U_2^3} > 0$$

We can observe that diminishing marginal utility ( $U_{ii} < 0$ ) is not necessary for convexity since  $U_{12}$  can be

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<sup>1</sup>Where the subscripts of U denote partial derivatives with respect to the indicated variables.

positive and large. It is not either sufficient since  $U_{12}$  can be negative and large.<sup>1</sup> It is interesting at this moment to point out that Edgeworth probably did not notice this property of his generalized utility function since he continued to assume diminishing marginal utility for all commodities.<sup>2</sup>

Moreover, even with convexity, the generalized utility function no longer has the implication that all Engel curves have positive slopes, which allows for the mathematical understanding of inferior goods. The first-order conditions for utility maximization given a budget constraint are:

$$\frac{U_1}{U_2} = \frac{P_1}{P_2} \quad \text{and} \quad Y = P_1 x_1 + P_2 x_2$$

where  $P_1$  and  $P_2$  are the money prices, and  $x_1$  and  $x_2$  are

---

<sup>1</sup>In the additive case, where  $U_{12} = 0$ , at most one marginal utility can be increasing so that diminishing marginal utility for each commodity is not necessary.

<sup>2</sup>Stigler, "The Development of Utility Theory," p. 323.



the quantities to be purchased of each good at those prices, given  $Y$ , the total money income available for expenditure.

Differentiating these equations with respect to  $Y$  and solving the system, we obtain:

$$\frac{dX_2}{dY} = \frac{P_2 U_{11} - P_1 U_{12}}{P_2^2 U_{11} - 2P_1 P_2 U_{12} + P_1^2 U_{22}}$$

The denominator of the right hand side is negative if the indifference curves are convex to the origin. The numerator, however, can be positive with  $U_{12} < 0$ , so the whole expression may be negative and  $X_2$  may be inferior, allowing for a negatively sloped Engel curve.

With the additive utility function  $U_{12} = 0$ , and assuming  $U_{ii} < 0$ , the expression must be positive and both commodities must be normal.

Equally true, if we differentiate the above first-order conditions with respect to  $P_2$  (holding  $P_1$  and  $Y$  constants) and solve the system, we obtain:



$$\frac{dx_2}{dP_2} = \frac{P_1 U_1 + x_2 P_1 U_{12} - x_2 P_2 U_{11}}{P_2^2 U_{11} - 2P_1 P_2 U_{12} + P_1^2 U_{22}}$$

The denominator is negative for convexity, and the numerator may be negative if  $U_{12}$  is negative, so the expression may be positive, allowing therefore for the case of positively sloped ordinary demand curves.

With the additive utility function and diminishing marginal utility the expression must be negative.

## APPENDIX II.

CLASSICAL DEMAND CURVES AT  
VARIOUS LEVELS OF INCOME

The purpose of this appendix is to help us understand the relations between the slopes of Engel curves and the shifts of classical demand curves at various levels of income.

The General Cases of  
Normal and Inferior Goods

Figure 18A illustrates a set of indifference curves for a certain individual ( $U_0 < U_2 < U_3$ ), in a two-commodity world ( $X_A$  and  $X_B$ ). The initial budget constraint is  $MN$ , for a given level of money income  $Y_0$ . As the price of  $X_A$  decreases from  $P_0$  to  $P_1$ , the budget constraint shifts from  $MN$  to  $MQ$ , and money income is unchanged. In this way, we are able to derive a classical demand curve,  $D_0 = D_0(P; Y_0)$ , given money income at  $Y_0$ , as shown in

Figure 18. Derivation of the Engel curve  
for the general case of a normal  
good

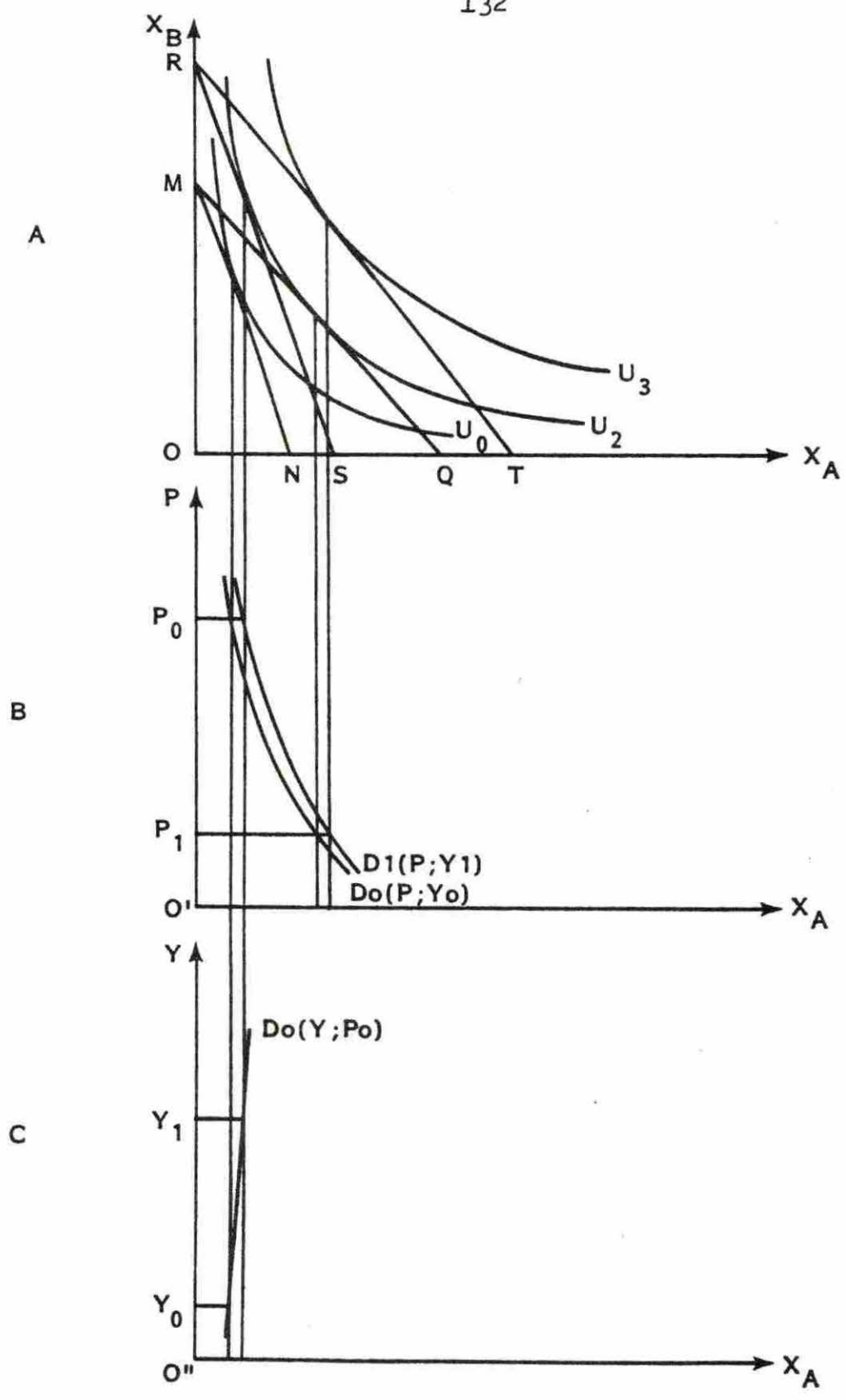


Figure 18B.

Suppose now that money income is raised to  $Y_1$ . This will cause the budget constraint at the initial relative price to be RS; and for the same decrease in  $P$ , it will be RT. A new demand curve  $D_1 = D_1(P; Y_1)$  is derived given money income  $Y_1$ .

It is obvious that commodity A is normal in this range of prices and income, and we verify that as money income goes up, the demand curve shifts to the right.

Holding price constant at  $P_0$ , we can determine how much of A will be demanded at each level of income. Figure 19C shows the Engel curve  $D_0 = D_0(Y; P_0)$  derived in this manner.<sup>1</sup>

This same procedure can be repeated for an inferior commodity, as Figure 19 illustrates.

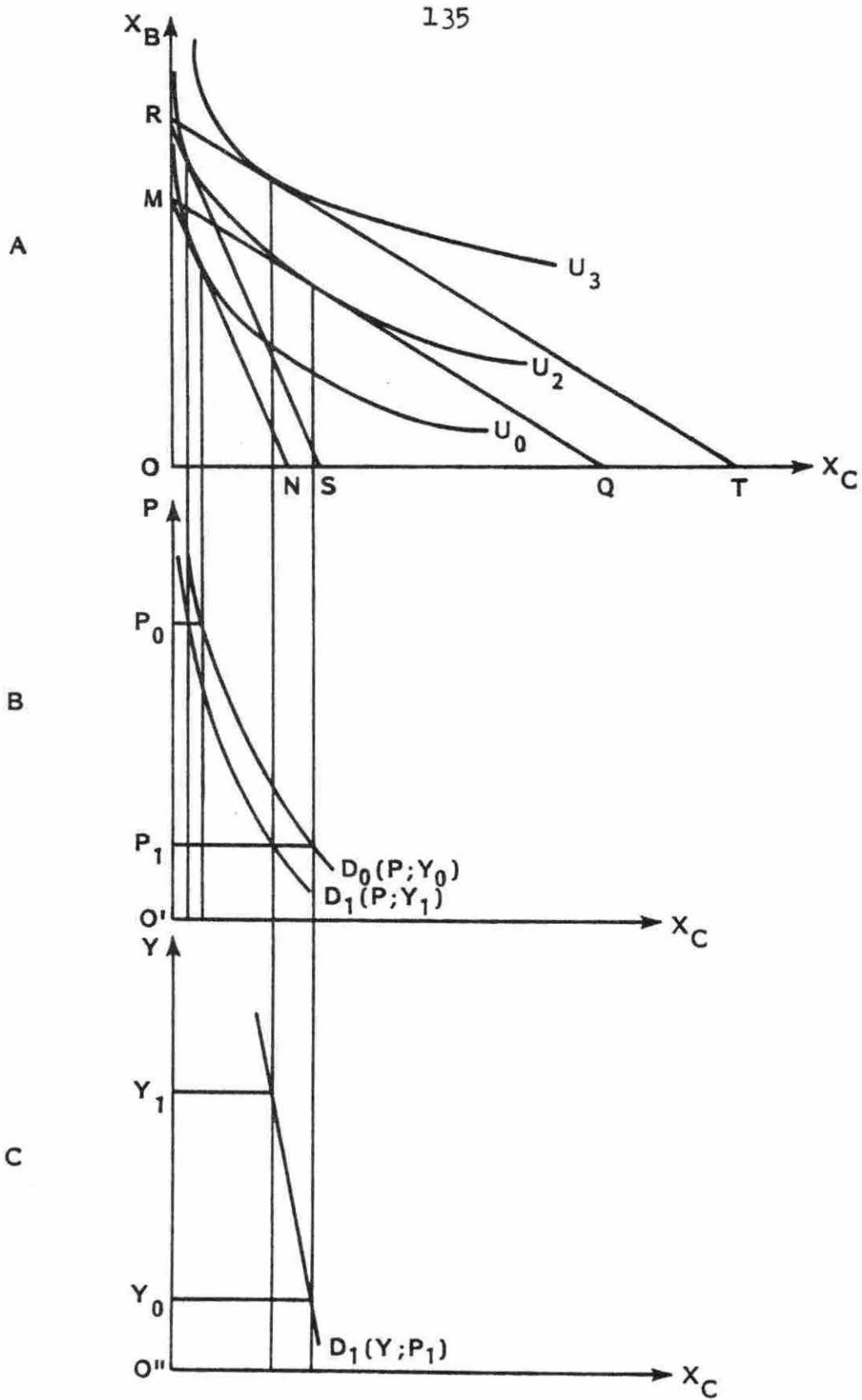
Hence, we conclude that, in general, an expansion of demand curves to the right, as money income increases,

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<sup>1</sup>This Engel curve and also those in Figures 19C and 20B are presented as a straight line for simplicity, since we are only interested in the sign of their slopes.

Figure 19. Derivation of the Engel curve for  
the general case of an inferior good





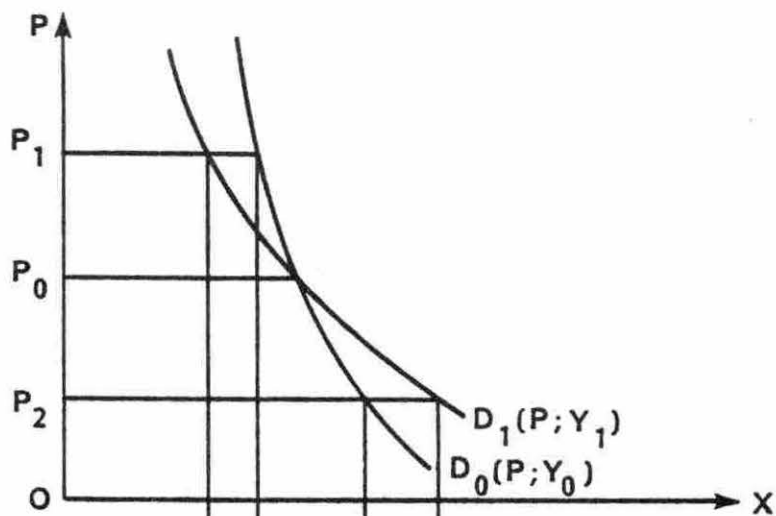
reflects the general case of a normal good, and that an expansion of demand curves to the left reflects the general case of an inferior good. We call these general cases because at those ranges of price and income we can say that commodities A and C are uniquely normal and inferior, respectively. We might have, however, cases when a good is not uniquely normal (or inferior) at all price levels, or at all income levels.

Goods not Uniquely Normal (Inferior)  
at All Price Levels

Figure 20A shows two demand curves for the same commodity at different levels of income ( $Y_0 < Y_1$ ) which cross each other at price level  $P_0$ . We can see that for prices above  $P_0$  the commodity in question behaves as an inferior good; and for prices below  $P_0$ , it behaves as a normal good. Hence, when prices above  $P_0$  are held constant, the Engel curves for this commodity will be negatively sloped, whereas when prices below  $P_0$  are held constant, the Engel curves will have a positive slope (see

Figure 20. The case of a good not uniquely  
normal (inferior) at all price levels

A



B

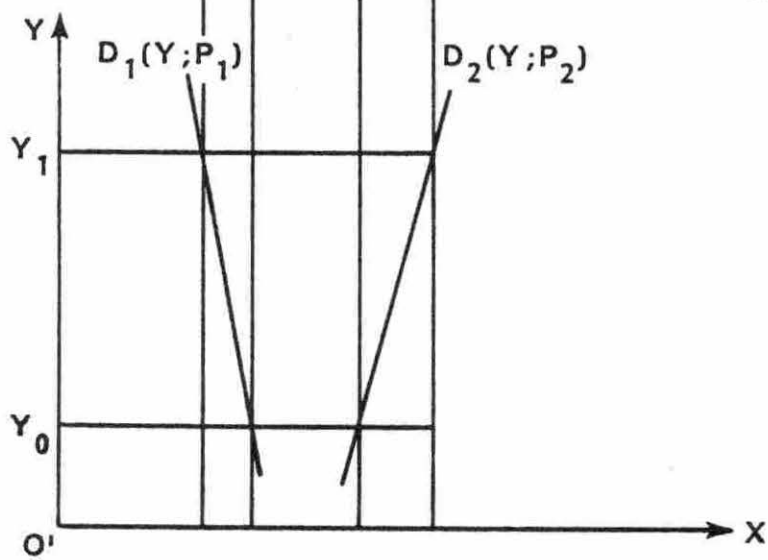


Figure 20B).

We conclude that any crossing of demand curves at different levels of money income imply reversal in the sign of the income elasticity of demand for that product.

#### Goods not Uniquely Normal (Inferior)

##### at All Income Levels

This is another special case. Suppose that there is a commodity whose Engel curve may reverse its slope (at the same price level). Assume that this good is normal at lower levels of income and is inferior at higher levels of income. Its demand curves will shift to the right with increases in income up to a certain point, and after that, additional increases in income will make the demand curves shift back to the left.

Now, restricting ourselves to the general case of a normal good (positive income elasticity of demand), let us investigate the cases of unitary elasticity of demand, income-elastic, and income-inelastic goods.

## Goods with Unitary Income

## Elasticity of Demand

To say that a commodity has unitary income elasticity of demand at a certain price level is the same as to say that the percentage increase in quantity demanded divided by the percentage increase in income equals unity. This implies that the Engel curve at that price level is a straight line through the origin. Figure 21 represents the case where unitary income elasticity of demand holds at all price levels.<sup>1</sup>

However, it is not necessary that unitary income elasticity of demand holds at all price levels. We may have unitary income elasticity only at price level  $P_0$ , as in Figure 22, and have the good be income-elastic above  $P_0$ , and be income inelastic below  $P_0$ .

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<sup>1</sup>Note that in Figure 22  $Y_0 < Y_1 < Y_2 < Y_3$  with equal increments. The same will apply for Figures 22, 23, and 24.



Figure 21. The case of a normal good with unitary income elasticity of demand at all price levels

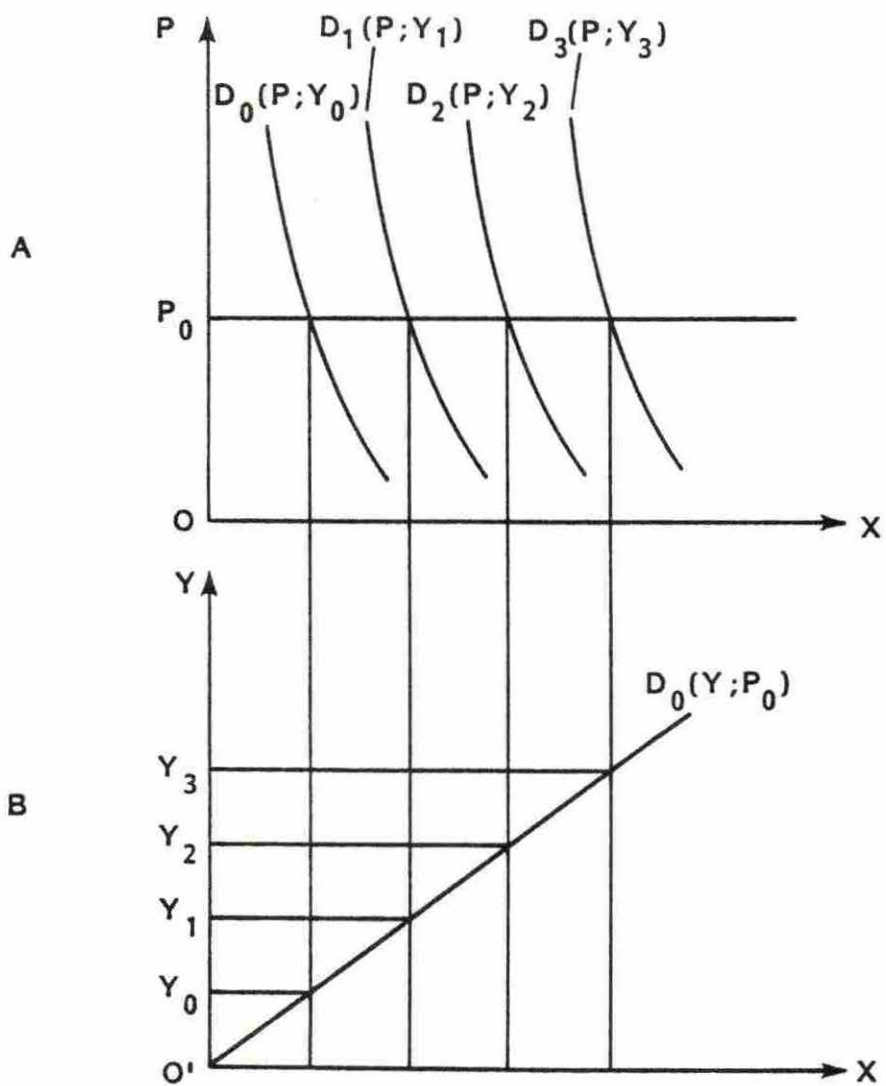
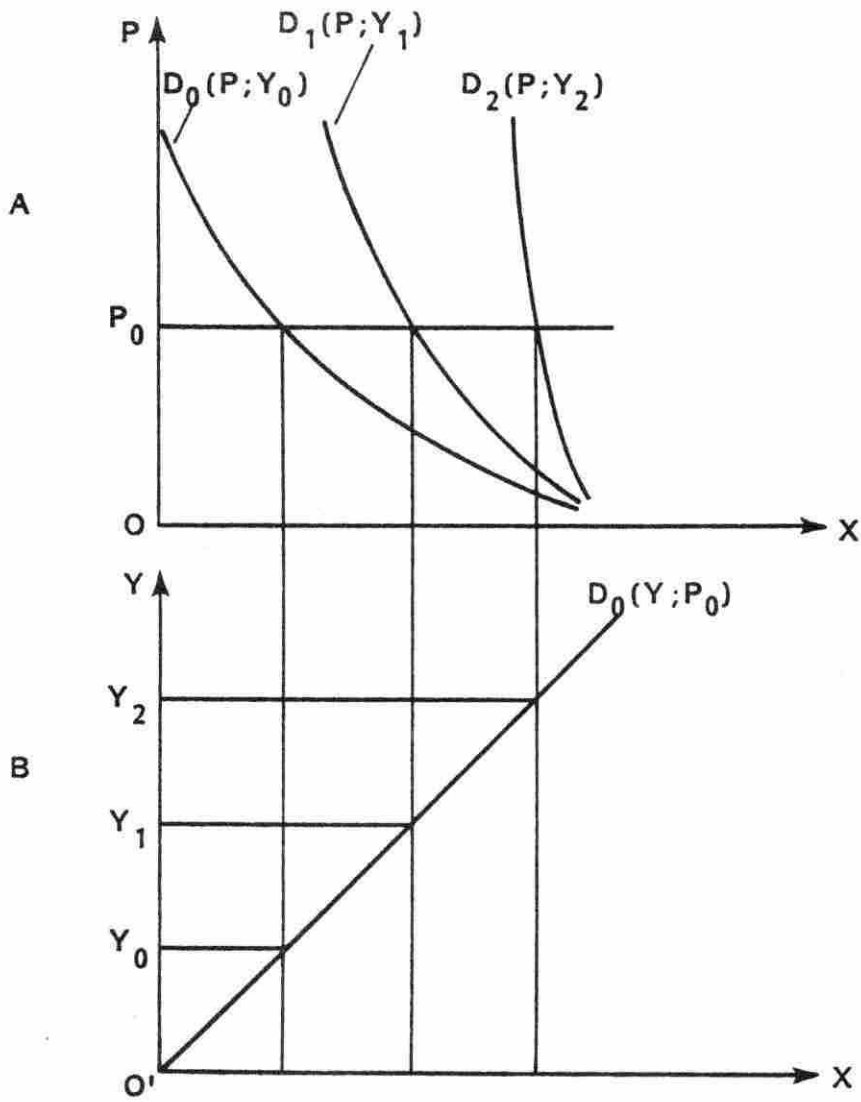


Figure 22. The case of a normal good with unitary income elasticity of demand at only one price level



### Income-Elastic Goods at All Prices

We may see also the case when the good has income elasticity of demand greater than one at all price levels. This implies that the Engel curves at all price levels will have a positive (for the normal good), but diminishing slope, as in Figure 23.

### Income-Inelastic Goods at All Prices

Or we may see the case when the good has income elasticity of demand less than one at all price levels. This implies that the Engel curves at all price levels will have a positive (for a normal good) and increasing slope, as in Figure 24.

Figure 23. The case of an income-elastic  
good at all price levels



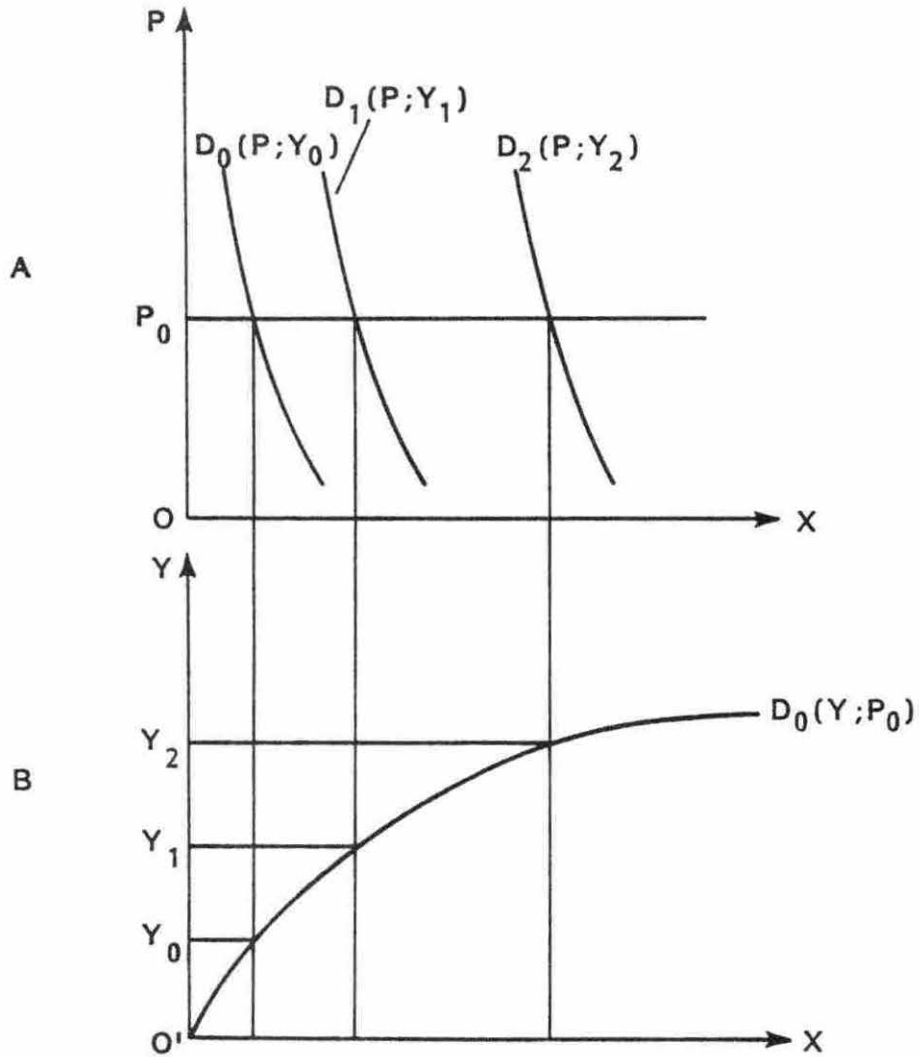
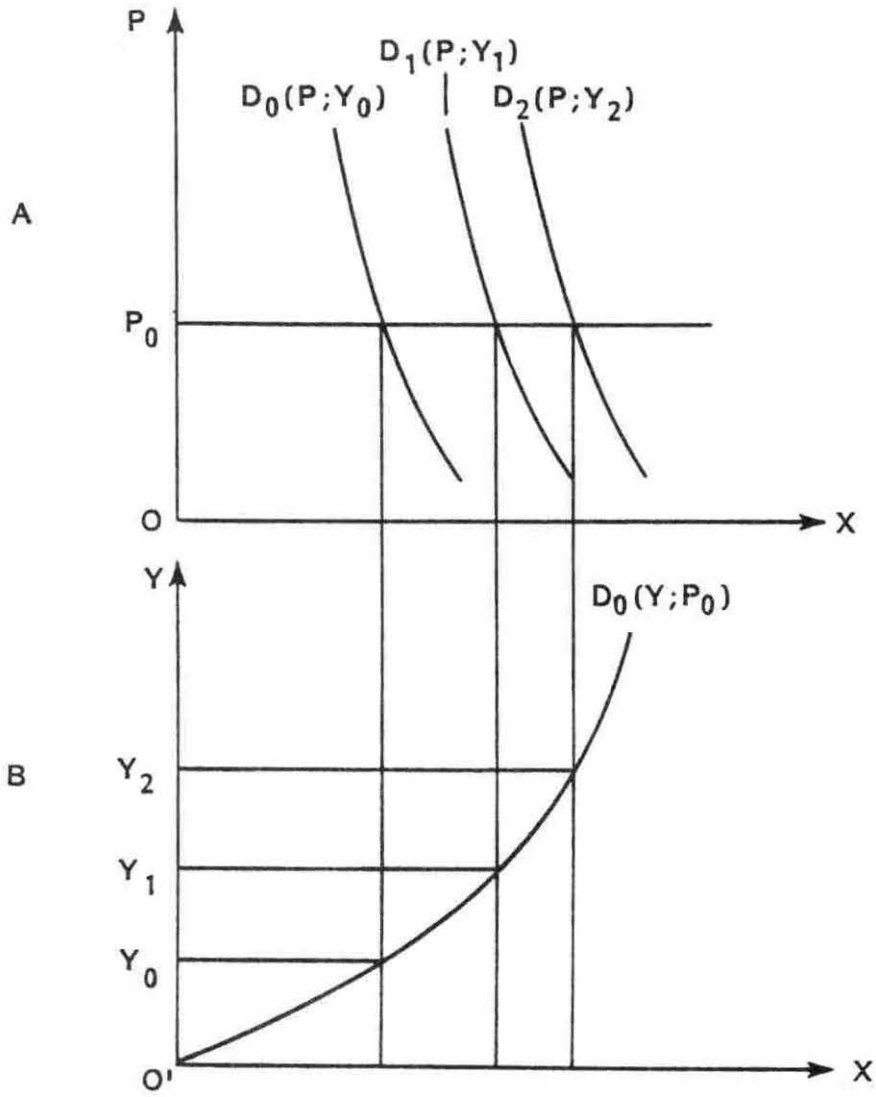


Figure 24. The case of an income-inelastic good at all price levels



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